Delay Constrained Energy Minimization in UWB Wireless Networks

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***Abstract*—We study the optimal power control, rate adaptation and scheduling for energy minimization subject to delay, traffic demand, transmit power and SNIR constraints in Ultra-Wideband wireless networks. We first show that power control is not required for delay constrained energy minimization. We then formulate optimal scheduling problem as an exponential size Linear Programming (LP) problem for which we propose the Pricing Minimization based Column Generation Method (PM-CGM). PM-CGM decomposes the exponential size LP problem into two sub-problems Restricted Master Problem (RMP) and Pricing Problem (PP) and solves it iteratively. We solve the corresponding delay minimization problem for the initialization of the RMP and propose a pricing minimization based polynomial time algorithm to solve the non-linear integer PP formulation. Simulations illustrate that PM-CGM algorithm decreases the runtime required to solve the large scale LP problem considerably while performing very close-to-optimal for different network scenarios.**

***Index Terms*—Scheduling, energy minimization, delay minimization, power control, rate adaptation, UWB.**

I. Introduction

Ultra-wideband (UWB) is a radio technology used for transmission of data spread over a bandwidth more than either 500MHz or 20% of the center frequency. This ultra wide bandwidth achieves robust communication at very low energy level and high data rate since it resists to multi-path fading, interference and decreases the power loss due to the lack of line-of-sight in very harsh environments.

The scheduling algorithms designed for UWB wireless networks mostly aim at maximizing throughput along with considering fairness among the wireless nodes since they usually consider multimedia applications requiring high-rate data transfer [1], [2], [3], [4]. However, the objective of maximizing throughput cannot be considered to be applicable for time-critical UWB applications where the objective is delay constrained energy minimization given the data traffic, transmit power and SNIR requirements on the links.

Scheduling for delay constrained systems have been investigated for interference-free and interference-controlling communication schemes. For interference-free scheduling, the studies aim at minimizing energy consumption by determining the optimal packet transmission times and durations where all packets have a common deadline [5], [6] or individual deadlines [7], [8]. The main finding of these studies which is minimization of energy consumption by transmitting the packets in longest possible duration however is not applicable to short range transmissions since the energy consumption due to circuitry during data transmission dominates the energy consumption due to actual data transmission which is shown to be valid for UWB transmissions in [9]. On the other hand, the scheduling algorithms designed for interference-controlling communication schemes aim at determining the best assignment of simultaneous transmissions considering optimal power allocation [10], [11], but not exploiting the rate adaptation for energy minimization. Delay constrained energy minimization has been investigated in a joint framework of optimizing transmission rates, powers and scheduling only for narrowband long-range wireless networks [12].

Determining the best assignment of simultaneous transmissions for optimal scheduling for delay constrained energy minimization necessitates solving very large scale problems since the number of possible concurrently active link sets is exponential in the number of links. Such a large scale problem can be solved using Column Generation Method (CGM) [13] which decomposes it into master and pricing sub-problems and solves rapidly in an iterative scheme. In the context of scheduling in wireless networks, CGM has been investigated for throughput maximization in [14], [15] and delay minimization in [16] where the authors have not considered rate adaptation. In [17], we have extended the delay minimization formulation presented in [16] to solve the minimum-length scheduling problem in UWB wireless networks considering the adaptability of transmission rate of a link to the SNIR level achieved at the receiver to meet certain bit/packet error rate requirement.

The aim of this study is to determine the optimal power control, rate adaptation and scheduling for the objective of minimizing energy subject to delay, data traffic, reliability and transmit power constraints on the links in UWB wireless networks. The main contributions are as follows. First, we determine optimal power and rate allocation for energy minimization with delay constraint in UWB wireless networks. Second, we formulate the delay constrained energy minimization problem as a very large scale Linear Programming (LP) problem with exponential number of variables in the number of the links. Third, we propose PM-CGM scheduling algorithm that decomposes the exponential size LP problem into two sub-problems called Restricted Master Problem (RMP) and Pricing Problem (PP). Fourth, we exploit the corresponding delay minimization problem to find an initial delay-feasible schedule for initialization of RMP and propose a heuristic method replacing the intractable optimal PP formulation. Simulations illustrate that PM-CGM decreases the runtime considerably with respect to the optimal LP formulation while achieving very close to optimal solutions.

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II. Joint Scheduling, Power and Rate Allocation Problem for Energy Minimization

The optimal power control, rate adaptation and scheduling problem for energy minimization in UWB wireless networks subject to total delay, link traffic demand, transmit power and SNIR constraints is formulated as follows:

minimize

$$\sum_{n=1}^{N}\sum_{l=1}^{L} p_l^{(n)} t^{(n)} + \sum_{n=1}^{N}\sum_{l=1}^{L} 1_{\left\{p_l^{(n)}>0\right\}} (p_{tx}+p_{rx}) \tag{1}$$

subject to

$$\sum_{n=1}^{N} t^{(n)} \leq D_{max} \tag{2}$$

$$\sum_{n=1}^{N} t^{(n)} x_l^{(n)} \geq R_l, \quad l \in [1,L] \tag{3}$$

$$p_l^{(n)} \leqslant p_{max}, \quad l \in [1,L], n \in [1,N] \tag{4}$$

$$x_l^{(n)} \leq K \frac{p_l^{(n)} h_{ll}}{\beta_l \left(N_0 + \sum_{k=1,k\neq l}^{L} p_k^{(n)} h_{kl}\gamma\right)}, \quad l \in [1,L], n \in [1,N] \tag{5}$$

$$a_{lk} + 1_{\left\{p_l^{(n)}>0\right\}} + 1_{\left\{p_k^{(n)}>0\right\}} \leq 2, \quad l,k \in [1,L], n \in [1,N] \tag{6}$$

where N is the number of time slots, L is the number of links, D_{max} is the maximum delay requirement; i.e., maximum schedule length, R_l is the data requirement of link l, p_{max} is the maximum transmit power due to UWB regulations, p_{tx} and p_{rx} are the constant powers dissipated at the transmitter and receiver respectively during data transmission in active mode, K is a constant representing the linear mapping between SNIR level at the receiver and the achievable transmission rate, N_0 is the background noise, h_{kl} is the power gain from the transmitter of link k to the receiver of link l, γ is a UWB parameter depending on the pulse repetition time and shape, β_l is the SNIR threshold value for link l depending on the desired bit/packet error rate and modulation schemes, and a_{lk} is a constant with value 1 if two links l and k share a common wireless node and 0 otherwise. The variables are $p_l^{(n)}$, the transmission power of link l in time slot n; $x_l^{(n)}$, the transmission rate of link l in time slot n and $t^{(n)}$, the length of time slot n.

The objective of the optimization problem is to minimize the total energy consumption in the network. Equation (2) states the maximum delay constraint for the schedule to be constructed. Equations (3) and (4) represent the traffic demand and maximum transmission power constraints of the links respectively. Equation (5) is the achievable rate formulation for the links based on the rate adaptivity characteristic of UWB such that the transmitter of a link can adapt its transmission rate linearly to the SNIR level achieved at the receiver due to very large bandwidth. Equation (6) states that a particular node in the network can be either the transmitter or receiver end of, at most, one active link in a time slot.

Since it is hard to solve this general non-convex programming problem in this joint optimization framework, it is beneficial to solve optimally for each set of variables independently.

III. Optimal Rate and Power Allocation

For a fixed power allocation, using maximum achievable rate satisfying Equation (5) is optimal since increasing the transmission rate decreases the time duration required for the transmission of a fixed amount of data, which consequently decreases both the delay and energy consumption of a link. More important issue for the foregoing joint optimization problem is optimal power allocation since power allocation of a specific link interests all other links by means of interference it creates.

It is shown in [1] that any feasible rate allocation or average power consumption can be achieved with $0/p_{max}$ allocation. However, it is not obvious that the $0/p_{max}$ allocation is the optimum choice in delay constrained energy minimization problem. Suppose that we have achieved a schedule for minimizing energy while satisfying the delay constraint with an arbitrary power allocation. According to [1], it is possible to achieve the same delay with a new schedule in which the power allocation is limited to $0/p_{max}$ allocation. It is also possible to achieve the same energy consumption with a new schedule in which the power allocation is limited to $0/p_{max}$ allocation. However, schedules constructed with $0/p_{max}$ allocation for the aforementioned purposes are not necessarily the same, i.e., while we are constructing a new schedule based on $0/p_{max}$ power allocation in order to achieve the same delay, energy consumption may increase and vice versa. For this reason, we state the optimality of power allocation for delay constrained energy minimization.

Theorem 1: In the optimal solution for the delay constrained energy minimization problem described in Section II, each link is either active with maximum transmission power or inactive in a time slot.

Proof: Suppose that an arbitrary schedule satisfying the constraints in the delay constrained energy minimization problem is constructed for a network. Let a be the duration of a time slot in which links $\{1,2,..,m\}$ are active with corresponding power and rate allocations $\{p_1,p_2,..,p_m\}$ and $\{x_1,x_2,..,x_m\}$ respectively. Let f_k denote the number of data bits transmitted by link $k \in [1,m]$ in this time slot. Let E be the total energy consumption in this time slot. We will show that we can separate this time slot into two time slots such that when any link with an arbitrary power level is assigned to transmit power p_{max} and 0 in the first and second slot respectively while keeping the transmit power of the remaining links the same, the time duration of the first and second time slots denoted by $a^{(1)}$ and $a^{(2)}$ respectively required for the transmission of total f_k data bits by every link $l \in [1,m]$ results in total delay less than a, i.e. $a^{(1)} + a^{(2)} \leqslant a$, and energy consumption less than E.

First, we will show that $a^{(1)} + a^{(2)} \leqslant a$. Assume that in the first time slot the transmit power p_i of an arbitrary link i is assigned to p_{max} while keeping the transmit powers of the remaining links the same. The data rate of link i is $x_i^{(1)} = (p_{max}/p_i)\, x_i$ whereas the data rate of link j where $j \neq i$ is given by

$$x_j^{(1)} = \frac{N_0 + U_j + p_i C}{N_0 + U_j + p_{max} C} x_j \tag{7}$$

where U_j denotes the interference of the links except link i on link j and $C = h_{ij}\gamma$. Here, it is obvious that only link i satisfies its data requirement in the first time slot since $x_i^{(1)} \geq x_i$ and $x_j^{(1)} \leq x_j$ where $j \neq i$ therefore $a^{(1)} = (p_i/p_{max})\, a$. In order to satisfy the data requirements of the links other than link i, these links are allocated in the second time slot. The data rate of link j where $j \neq i$ in the second time slot is given by

$$x_j^{(2)} = \frac{N_0 + U_j + p_i C}{N_0 + U_j} x_j \tag{8}$$

For an arbitrary link j to satisfy its data requirement, the equation given as

$$a x_j = a^{(1)} x_j^{(1)} + a_j^{(2)} x_j^{(2)} \tag{9}$$

should be satisfied where $a_j^{(2)}$ denotes the actual amount of time required for link j to meet its data requirement. Substituting $a^{(1)}$ with $(p_i/p_{max})\, a$, $a_j^{(2)}$ is given by

$$a_j^{(2)} = a\{\frac{N_0 + U_j}{N_0 + U_j + p_i C} - \frac{p_i}{p_{max}} \frac{N_0 + U_j}{N_0 + U_j + p_{max} C}\} \tag{10}$$

Since we want all the links to satisfy their data requirements, $a^{(2)} = \max_{j \in [1,m], j \neq i} a_j^{(2)}$. Suppose that this maximum is achieved for link k. Then, the total length of $a^{(1)}$ and $a^{(2)}$, say $a^{'}$, is given by

$$a^{'} = a\{1 - \frac{p_i C}{N_0 + U_k + p_i C} + \frac{p_i C}{N_0 + U_k + p_{max} C}\} \tag{11}$$

It is clear that $a^{'} \leq a$ for every pair i,k and equality holds only if $p_i = p_{max}$. We can conclude that any link with an arbitrary power level can be allocated with 0/ p_{max} transmit power allocation resulting in a new schedule with less delay.

Now, we will investigate the energy consumption. Energy consumption in time slot of duration a is given by

$$E = a \sum_{j=1}^{m} \{p_j + p_{tx} + p_{rx}\} \tag{12}$$

Let $E^{(1)}$ and $E^{(2)}$ denote the energy consumption in the first and second time slots respectively. $E^{(1)}$ and $E^{(2)}$ are formulated as follows:

$$E^{(1)} = a^{(1)} \sum_{j=1}^{m} \{p_j + p_{tx} + p_{rx}\} + a^{(1)}\{p_{max} - p_i\} \tag{13}$$

$$E^{(2)} = a^{(2)} \sum_{j=1}^{m} \{p_j + p_{tx} + p_{rx}\} - a^{(2)} p_i \tag{14}$$

The total energy consumption, say $E^{'}$, is then given by

$$E^{'} = a^{'} \sum_{j=1}^{m} \{p_j + p_{tx} + p_{rx}\} + a^{(1)} p_{max} - \{a^{(1)} + a^{(2)}\} p_i \tag{15}$$

Substituting p_{max} with $p_i a/a^{(1)}$, we get

$$E^{'} = a^{'} \sum_{j=1, j\neq i}^{m} \{p_j + p_{tx} + p_{rx}\} + a p_i + a^{'}\{p_{tx} + p_{rx}\} \tag{16}$$

Since $a^{'} \leq a$, it follows that $E^{'} \leq E$ and equality holds only if $p_i = p_{max}$. □

IV. Optimal Scheduling

Given the optimal rate and power allocation provided in Section III, the joint optimization problem presented in Section II can be reduced to a pure scheduling problem. However, since we do not know the sets of concurrently transmitting links prior to scheduling, we need to consider all possible subsets of links for concurrent transmission as described next.

Let $\mathcal{E} = \{\mathcal{E}_k : 1 \leq k \leq |\mathcal{E}|\}$ denote the set of all feasible subsets of the link set $\mathcal{L} = \{1, 2, ..., L\}$. Note that $|\mathcal{E}|$ is equal to 2^L if any two links do not share a common node. Let $\mathbf{X}$ be an $L \times |\mathcal{E}|$ transmission rate matrix, where the element x_{lk} of $\mathbf{X}$ is the optimal transmission rate of link l in link set $\mathcal{E}_k$ given as

$$x_{lk} = \frac{p_{max} h_{ll}}{\beta_l} \left(N_0 + \sum_{i \in \{\mathcal{E}_k / l\}} p_{max} h_{il} T_f \gamma \right)^{-1} \tag{17}$$

Notice that each column of the matrix $\mathbf{X}$ gives the optimal transmission rates of the concurrently active links in a subset. Furthermore, let $\mathbf{e}$ denote a $1 \times |\mathcal{E}|$ vector whose k-th element is equal to the number of active links in link set $\mathcal{E}_k$.

Then, optimal scheduling problem for energy minimization with delay constraint can be formulated as a Linear Programming (LP) problem as:

minimize

$$(p_{max} + p_{tx} + p_{rx})\mathbf{et} \tag{18}$$

subject to

$$\mathbf{Xt} \geq \mathbf{R} \tag{19}$$

$$-\mathbf{1t} \geq -D_{max} \tag{20}$$

where $\mathbf{1}$ is a $1 \times |\mathcal{E}|$ all-ones vector and $\mathbf{R}$ is a $L \times 1$ vector containing the traffic demand requirements of the links, i.e. $\mathbf{R} = [R_1, R_2, ..., R_L]$. The variable of the problem is $|\mathcal{E}| \times 1$ vector $\mathbf{t}$ whose k-th element is the time slot duration allocated for the subset $\mathcal{E}_k$. Equations (19) and (20) represent the traffic demand requirements of the links and delay requirement of the schedule respectively.

There are two difficulties in solving this optimization problem although it is an LP problem. An exponential time effort is required to form $\mathbf{X}$ matrix and solve the corresponding LP since the number of link subsets so the number of variables is exponential in the number of links. These difficulties can be overcome by reducing the size of $\mathcal{E}$ thus dealing with a small size rate matrix $\mathbf{X}$.

V. Price Minimization Based Column Generation Method (PM-CGM)

The intractability of the exponential size LP formulation given in Section IV can be overcome by using Column Generation Method (CGM). In CGM, the large scale original LP problem is decomposed into a Restricted Master Problem (RMP) and a Pricing Problem (PP) and the original problem can be solved in an iterative way. We start with an initial feasible RMP which is a restricted; i.e., small-scale, version of the exponential LP formulation. Then, we pass the dual solution of RMP to PP. If the optimal solution of the PP can be used to improve the solution of the RMP, we pass the vector corresponding to the optimal solution to RMP by adding it to the constraint matrix of RMP as a column. Then RMP is solved again and its dual is passed to PP to make PP generate another column. This iterative behaviour continues until the optimal solution of the PP cannot be used to improve the solution of the RMP. This column generation idea is the center of PM-CGM algorithm.

A. Restricted Master Problem (RMP)

We reduce the original problem to the following restricted problem given as

minimize

$$\boldsymbol{e}^s \mathbf{t} \tag{21}$$

subject to

$$\boldsymbol{X}^s \mathbf{t} \geq \mathbf{R} \tag{22}$$

$$-\mathbf{1t} \geq -D_{max} \tag{23}$$

The difference between the original problem and the foregoing restricted problem is that only a subset of $\mathcal{E}^s \subset \mathcal{E}$ is considered in the restricted problem and $\boldsymbol{X}^s$ and $\boldsymbol{e}^s$ denote the transmission rate matrix and the vector of numbers of active links corresponding to the subset $\mathcal{E}^s$ respectively. Note that we have removed the constant multiplier $(p_{max} + p_{tx} + p_{rx})$ in the objective since it does not affect the optimal solution. For the first iteration of PM-CGM, we need an initial transmission rate matrix $\boldsymbol{X}^s$ that guarantees a feasible solution for the RMP; i.e, both data requirements of the links and total delay requirement for the schedule can be satisfied. However, this is not a straightforward problem. For now, we assume that such an initial matrix is available. We will discuss this in Section V-C.

RMP can be solved using simplex method in polynomial time and primal optimal solution $\boldsymbol{t}^p$ and dual optimal solution $\boldsymbol{t}^d$ corresponding to the dual of RMP can be obtained. Note that $\boldsymbol{t}^p$ may not be the optimal solution of the original problem; i.e., $\boldsymbol{t}^p$ is the optimal solution only when $\mathcal{E}^s$ contains the optimal sets. The reduced cost of a column in the original constraint matrix of the problem given by Equations (18-20) but not in the restricted constraint matrix of the problem given by Equations (21-23) is

$$c_k = e_k - (\sum_{i=1}^{L} t_i^d x_{ik}) + t_{L+1}^d \tag{24}$$

where t_l^d is the l-th element of the optimal dual solution of the RMP. Negativity of the reduced cost of any column in the original constraint matrix implies that the optimal solution of the RMP can be improved; i.e. decreased, by adding the column with the negative reduced cost to the restricted constraint matrix. If there is not a column with negative reduced cost, the solution $\boldsymbol{t}^p$ of the RMP is the optimal solution to the original large scale problem. In order to find if there is such a column, one needs to solve the pricing problem discussed next.

B. Pricing Problem (PP)

Pricing Problem that is used to generate a column with the minimum reduced cost is formulated as

minimize

$$\sum_{l=1}^{L} b_l - \sum_{l=1}^{L} t_l^d K \frac{b_l p_{max} h_{ll}}{\beta_l \left(N_0 + \sum_{k=1,k\neq l}^{L} b_k p_{max} h_{kl} \gamma\right)} + t_{L+1}^d \tag{25}$$

subject to

$$a_{lk} + b_l + b_k \leq 2, \quad l,k \in [1,L] \tag{26}$$

$$b_l \in \{0,1\}, \quad l \in [1,L] \tag{27}$$

The variables of the optimization problem are b_l for each link l, which acts as an indicator for activity of link l; i.e., takes value 1 if link l is active and 0 otherwise.

PP minimizes the reduced cost given by Equation (24) over all columns $\boldsymbol{x}^{(k)}$ of the original transmission rate matrix $\boldsymbol{X}$ subject to Equation (26) representing the availability of concurrent transmissions originally given in Equation (6) using integer variables b_l. After determining b_l for each link l, $\boldsymbol{x}^{(k)}$ is the column vector of maximum achievable rates given by Equation (5) with the assigned power allocation $b_l p_{max}$, i.e. $x_l = K\frac{b_l p_{max} h_{ll}}{\beta_l \left(N_0 + \sum_{k=1,k\neq l}^{L} b_k p_{max} h_{kl} \gamma\right)}$. Note that, one must append a (-1) to the column $\boldsymbol{x}^{(k)}$ before passing to the RMP to completely characterize a column in the constraint matrix due to the delay constraint represented by Equation (23). Moreover, cost coefficient of the variable corresponding to the vector passed to the RMP is $\sum_{l=1}^{L} b_l$.

The foregoing PP formulation is a non-linear integer programming problem which is NP-hard and cannot be solved in polynomial time. In order to deal with this complexity problem, we propose Price Minimization Algorithm (PMA) to solve PP fast and efficiently. We define the price $P_{\mathcal{S}}$ for a set $\mathcal{S}$ of links as

$$P_{\mathcal{S}} = |\mathcal{S}| - \sum_{l\in\mathcal{S}} t_l^d K \frac{p_{max} h_{ll}}{\beta_l \left(N_0 + \sum_{k\neq l, k\in\mathcal{S}} p_{max} h_{kl} \gamma\right)} + t_{L+1}^d \tag{28}$$

Price Minimization Algorithm (PMA)

1: $\mathcal{S} = \emptyset$; $\mathcal{S}' = \mathcal{L}$;
2: **while** $\mathcal{S} \neq \mathcal{L}$ **do**
3:    **if** $\min_{i\in\mathcal{S}'} P_{\mathcal{S}+\{i\}} < P_{\mathcal{S}}$ **then**
4:       k=argmin$_{i\in\mathcal{S}'} P_{\mathcal{S}+\{i\}}$;
5:       $\mathcal{S} = \mathcal{S} + \{k\}$;
6:       $\mathcal{S}' = \mathcal{S}' - \{k\}$;
7:    **else**
8:       **break**;

In each iteration, PMA algorithm picks one link to add to set $\mathcal{S}$ such that this link minimizes the price of the set after it is added to the set (Lines 4-5). The algorithm terminates either when the set $\mathcal{S}$ includes all the links in the link set $\mathcal{L}$ (Line 2) or the price of the set $\mathcal{S}$ cannot further decrease by the addition of a link (Line 3). If the price of the output set $\mathcal{S}$ returned by the PMA is less than 0, the vector of transmission rates corresponding to set $\mathcal{S}$, with a (-1) appended as the last element, is passed to the RMP since it decreases the objective of the RMP; i.e., the energy consumption. Otherwise, the PM-CGM algorithm terminates.

C. Determination of a Feasible Initial Matrix for RMP

PM-CGM algorithm requires an initial transmission rate matrix $\boldsymbol{X}^s$ that guarantees a feasible solution for the RMP as stated in Section V-A. In order to produce an $\boldsymbol{X}^s$ matrix that satisfies both data and delay requirements, we propose solving the delay minimization problem given as

minimize

$$\mathbf{1t} \tag{29}$$

subject to

$$\mathbf{Xt} \geq \mathbf{R} \tag{30}$$

Feasibility condition of the energy minimization problem with delay constraint is equivalent to the condition that the optimal solution of the above delay minimization problem is less than the delay requirement D_{max}. Hence, if we solve this delay minimization problem, the columns in $\mathbf{X}$ corresponding to the positive variables in the optimal solution $\mathbf{t}$ constitutes a feasible initial matrix $\boldsymbol{X}^s$ if the optimal delay is less than D_{max}. However, since this is again a large scale LP programming problem with exponential number of variables in the number of links, it is intractable. In [17], we have proposed a column generation based fast scheduling algorithm for the delay minimization problem with data constraints on the links. The solution procedure is directly applicable here. Note that we can stop at the particular iteration of the column generation that returns a schedule with delay less than D_{max} since it suffices to create a schedule with length less than or equal to the delay constraint D_{max}.

VI. Simulations and Performance Evaluation

Simulations are performed on a computer with a CPU of 2.5GHz processing speed and 4GB RAM in MATLAB. The path loss model used is $PL(d) = PL(d_0) - 10\alpha\log_{10}\left(\frac{d}{d_0}\right) + Z$ where d is the distance between the transmitter and receiver, d_0 is the reference distance with value 1m, $PL(d)$ is the path loss at distance d, α is the path loss exponent and Z is a Gaussian random variable with zero mean and σ_z^2 variance. The simulation parameters are given in Table-I.

α	4	σ_z^2	2	p_{max}	10mW
β_l	10 dB	$PL(d_0)$	30 dB	p_{tx}	30mW
K	10^6	N_0	10^{-8} W/Hz	p_{rx}	100mW

TABLE I: Simulation Parameters

For a better characterization of the algorithm performance, we have performed the simulations on 1000 independent random network topologies for each simulation scenario. As an alternative scheduling algorithm, we use Fixed Rate Allocation algorithm, denoted as FRA. In FRA algorithm, among all possible sets of links, the feasible ones are determined based on a fixed predetermined rate $\mathbf{r}$. The fixed transmission rate $\mathbf{r}$ is determined as the average of the minimum and the maximum transmission rates for which a feasible schedule can be constructed. The aim of using FRA algorithm for comparison is to illustrate the superiority of rate-adaptivity of PM-CGM over fixed rate allocation schemes.

Figure-1 illustrates the approximation ratio performance of PM-CGM and FRA algorithms for different number of links with 95% confidence intervals depicted around the means where approximation ratio is the ratio of the energy consumption achieved by the algorithm to the minimum energy consumption obtained by solving the exponential LP (EXP-LP) formulation given in Equations (18-20). The approximation ratio of PM-CGM scheduling algorithm is very close to 1; i.e. optimal solution, and outperforms FRA algorithm significantly for different number of links. In addition, the approximation ratio of PM-CGM is robust to the increase in the number of the links whereas the approximation ratio of the FRA algorithm increases as the number of links increases. The small lengths of the confidence interval bars also demonstrate the robustness of PM-CGM to different topologies.

Besides performing very close to optimal, PM-CGM decreases the running time required to solve EXP-LP to a greater extent as Figure-2 illustrates. The runtime of PM-CGM increases linearly whereas the runtime of EXP-LP increases exponentially with the number of links. The average runtime of PM-CGM is only 1% of the average runtime of EXP-LP for a network of 20 links. Note that the reason of performing simulations for up to 20 links is the intractability of EXP-LP for more than 20 links; i.e., it is not possible to solve EXP-LP in MATLAB.

Figure-3 illustrates the robustness of the performance of PM-CGM to varying network densities. The PM-CGM is closer to the optimal at low and high network densities that correspond to the cases where the links transmit all simultaneously and one by one respectively since optimality of concurrent transmissions

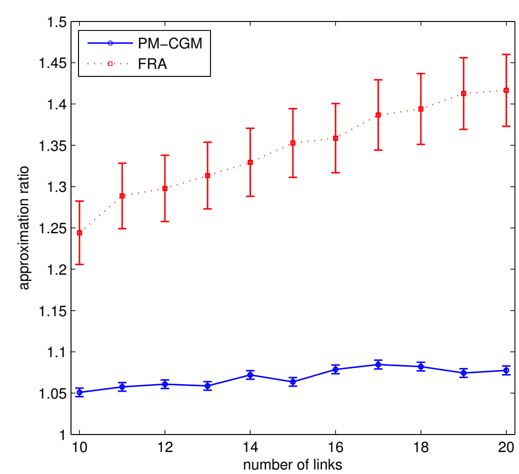

Fig. 1: Approximation ratio of PM-CGM and FRA algorithms for different number of links with 95% confidence intervals depicted around the means.

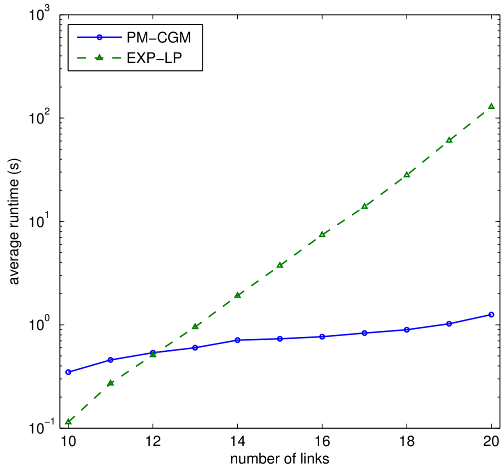

Fig. 2: Average running time comparison of PM-CGM and EXP-LP for different number of links.

is more obvious in these cases compared to medium network densities.

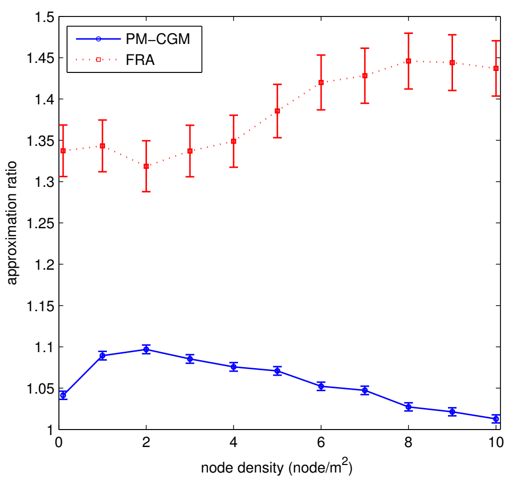

Fig. 3: Approximation ratio of PM-CGM and FRA algorithms for different node densities with 95% confidence intervals depicted around the means.

VII. Conclusion

In this paper, we study the optimal power control, rate adaptation and scheduling problem for delay constrained energy minimization in UWB wireless networks subject to total delay, link traffic demand, transmit power and SNIR requirements. Upon stating the optimal rate and power allocation analytically; i.e., each link is active with maximum transmit power and corresponding maximum achievable rate or inactive at a time at the optimal solution, we formulate the scheduling problem as a large scale LP problem of exponential size in the number of the links. We then propose the column generation based algorithm called Pricing Minimization based Column Generation Algorithm (PM-CGM) decomposing this exponential size LP problem into Restricted Master Problem (RMP) and Pricing Problem (PP) by exploiting the column generation technique used for solving large scale problems. For the initialization of the RMP problem, we illustrate that a feasible schedule can be generated by running the CGM on the corresponding delay minimization problem until delay-feasibility is satisfied. We propose a polynomial-time algorithm to replace the intractable PP formulation. Through simulations, we show that PM-CGM performs very close to the optimal for various network topologies containing different number of nodes at different densities and decreases the runtime required to solve the exponential LP formulation considerably.

References

[1] B. Radunovic and J. L. Boudec, "Optimal power control, scheduling, and routing in uwb networks," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 7, pp. 1252–1270, September 2004.

[2] K. H. Liu, L. Cai, and X. Shen, "Exclusive-region based scheduling algorithms for uwb wpan," *IEEE Transactions on Wireless Communications*, vol. 7, no. 3, pp. 933–942, March 2008.

[3] H. Jiang and W. Zhuang, "Effective packet scheduling with fairness adaptation in ultra-wideband wireless networks," *IEEE Transactions on Wireless Communications*, vol. 6, no. 2, pp. 680–690, February 2007.

[4] K. H. Liu, L. Cai, and X. Shen, "Multiclass utility-based scheduling for uwb networks," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 2, pp. 1176–1187, March 2008.

[5] E. Uysal-Biyikoglu, B. Prabhakar, and A. E. Gamal, "Energy-efficient packet transmission over a wireless link," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 487–499, August 2002.

[6] F. Zhang and S. Chanson, "Improving communication energy efficiency in wireless networks powered by renewable energy sources," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 6, pp. 2125–2136, November 2005.

[7] W. Chen and U. Mitra, "Energy efficient scheduling with individual packet delay constraints," in *IEEE INFOCOM*, June 2006, pp. 1–6.

[8] X. Zhong and C.-Z. Xu, "Energy-efficient wireless packet scheduling with quality of service control," *IEEE Transactions on Mobile Computing*, vol. 6, no. 10, pp. 1158–1170, October 2007.

[9] T. Wang, W. Heinzelman, and A. Seyedi, "Link energy minimization in ir-uwb based wireless sensor networks," *IEEE Transactions on Wireless Communication*, vol. 9, no. 9, pp. 2800–2811, September 2010.

[10] T. Elbatt and A. Ephremides, "Joint scheduling and power control for wireless ad hoc networks," *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, pp. 74–85, January 2004.

[11] L. Fu, S. C. Liew, and J. Huang, "Fast algorithms for joint power control and scheduling in wireless networks," *IEEE Transactions on Wireless Communications*, vol. 9, no. 3, pp. 1186–1197, March 2010.

[12] R. Madan, S. Cui, S. Lall, and A. J. Goldsmith, "Cross-layer design for lifetime maximization in interference-limited wireless sensor networks," *IEEE Transactions on Wireless Communications*, vol. 5, no. 11, pp. 3142–3152, November 2006.

[13] D. Bertsimas and T. Tsitsiklis, *Introduction to Linear Optimization*. Athena Scientific, 1997.

[14] M. Johansson and L. Xiao, "Cross-layer optimization of wireless networks using nonlinear column generation," *Wireless Communications, IEEE Transactions on*, vol. 5, no. 2, pp. 435 – 445, feb. 2006.

[15] J. Luo, C. Rosenberg, and A. Girard, "Engineering wireless mesh networks: Joint scheduling, routing, power control, and rate adaptation," *Networking, IEEE/ACM Transactions on*, vol. 18, no. 5, pp. 1387 –1400, oct. 2010.

[16] L. Fu, S. C. Liew, and J. Huang, "Fast algorithms for joint power control and scheduling in wireless networks," *IEEE Transactions on Wireless Communications*, vol. 9, no. 3, pp. 1186 –1197, March 2010.

[17] Y. Sadi and S. Ergen, "Fast scheduling for delay minimization in uwb wireless networks," *Communications Letters, IEEE*, vol. 16, no. 9, pp. 1400 –1403, september 2012.