Scheduling in Single-Hop Multiple Access Wireless Networks with Successive Interference Cancellation

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***Abstract*—We study the optimal scheduling problem for minimizing the length of the schedule required to satisfy the traffic demands of the links in single-hop multiple access wireless networks using Successive Interference Cancellation (SIC). We first formulate the optimal scheduling as a Linear Programming (LP) problem where each variable represents the time allocated to a subset of the links sorted according to their decoding order for SIC at the receiver. Since the LP formulation requires a high number of variables exponential in the decoding capability of the receiver, we propose a Column Generation Method (CGM) based heuristic algorithm. This algorithm is based on decomposing the original problem into Restricted Master Problem (RMP) and Pricing Problem (PP), and approximating the exponentially complex PP by a greedy heuristic algorithm. Compared to the CGM based heuristic algorithms previously proposed for minimum length scheduling problem in various types of networks, the formulation of the PP and the heuristic algorithm proposed for PP are different due to the requirement of including the decoding order of simultaneous transmissions in SIC based networks. We demonstrate via simulations that the proposed algorithm performs very close to the optimal LP formulation with runtime robust to the increasing number of the links and decoding capability of the receiver, and much smaller than that of the optimal algorithm.**

***Index Terms*—Scheduling, successive interference cancellation, wireless networks, optimization, column generation method.**

I. Introduction

INTERFERENCE avoidance has been commonly used to deal with wireless channel interference in the design of network protocols. In interference avoidance, a receiver can only decode one transmission at a time by considering all other transmissions as interference. The arrival of multiple transmissions at a receiver results in a collision so failure of reception. On the other hand, interference cancellation allows detecting multiple transmissions at a time by decomposing all the signals in a composite signal. Among many interference cancellation techniques, SIC appears to be the most promising due to its simplicity, overall system robustness [1] and existing prototypes [2]. SIC is based on decoding the signals from multiple transmitters successively. Each time a signal is decoded, it is subtracted from the composite signal to improve the Signal-to-Interference-plus-Noise Ratio (SINR) of the remaining signals.

Time Division Multiple Access (TDMA), where only one transmission is scheduled at a time, has often been used as a conflict-free scheduling in single-hop multiple access wireless networks with interference avoidance. Although TDMA is simple and easy to implement, it leads to suboptimal channel usage since it doesn't exploit the capability of multiple transmission detection. On the other hand, SIC based single-hop multiple-access wireless networks still require an efficient scheduling algorithm since in practice a receiver node may only decode a certain number of transmissions at a time [3].

The goal of this paper is to study the joint optimization of the scheduling algorithm and rate allocation to minimize the completion time required to satisfy the given traffic demands of the links, defined as the schedule length, in SIC based single-hop multiple-access wireless networks. The resource allocation problem has received a lot of attention for interference avoidance based wireless networks in the past. However, the optimization problems formulated for these networks cannot be adapted to SIC based networks due to the difficulty of including the SINR requirement considering the ordering of the SIC decoding as a constraint in the optimization problem [4], [5]. In SIC based networks, the SINR of a link depends on the decoding order of simultaneous transmissions eliminating the previously decoded ones and considering the later decoded ones as interference. Including all possible SIC decoding orderings is only possible via assigning a variable to every possible subset of the links sorted according to their decoding order for SIC at the receiver [6]. Since such a formulation requires a high number of variables exponential in the decoding capability of the receiver, a greedy heuristic algorithm has been proposed in [6]. However, this heuristic algorithm has no relation to the optimal formulation so performs much worse than the optimal solution.

In this letter, we propose a heuristic algorithm based on the CGM [7] to solve the large-scale optimization problem efficiently. This algorithm is based on decomposing the original problem into Restricted Master Problem (RMP) and Pricing Problem (PP), and generating the variables of the RMP iteratively based on a greedy heuristic algorithm that approximates the exponentially complex PP. CGM based heuristic methods have been proposed for power controlled fixed rate wireless networks [4] and adaptive rate Ultra-Wideband (UWB) networks [8]. Although the main structure of the RMP in our proposed algorithm is similar to that in [4], [8], the formulation of the PP and the heuristic algorithm proposed for PP are different due to the requirement of including the decoding order of simultaneous transmissions in SIC based networks. The main novelties of this paper are the proposal of using CGM in the scheduling of the SIC based networks, the proposal of the heuristic algorithm following the proof of the hardness of the CGM method and the demonstration of the superior performance of the proposed CGM based heuristic algorithm compared to the previously proposed methods.

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II. System Model and Assumptions

The system model and assumptions are detailed as follows:

- The wireless network consists of a set of L directed links corresponding to L transmitters and a common receiver node. Let us denote the link set by $\mathcal{L} = \{1, 2, ..., L\}$.
- The links in the set $\mathcal{L}$ have positive traffic demands. Let us call the $L \times 1$ demand vector F such that $F = [f_1, f_2, ..., f_L]^T$ where f_i is the traffic demand of the i-th link.
- The time is partitioned into slots of possibly variable lengths.
- We assume that the signal removal of SIC is perfect but the receiver can only decode a certain number of simultaneous transmissions, which is denoted by K, considering the practical limitations as shown in [3], [6], [9].
- The transmitters in the network transmit at the maximum power level p_{max}. The set of received powers is denoted by $\mathcal{P}$ such that $\mathcal{P} = \{P_1, P_2, ..., P_L\}$ where P_i is the received power corresponding to the transmission at maximum power level at the i-th link.

III. Successive Interference Cancellation (SIC)

Let us denote the set of active links in a time slot by S. Let $r_i(S)$ be the transmission rate of link $i \in S$ when simultaneously transmitting with the links in S. The capacity region of the multiple access channel determines the feasible rates of the links in the set and is defined as the closure of the convex hull of the rate vectors satisfying

$$\sum_{i \in S'} r_i(S') \leq W log_2(1 + \frac{\sum_{i \in S'} P_i}{N_0}) \tag{1}$$

for all $S' \subseteq S$, where W is the channel bandwidth and N_0 is the power of the background noise [10]. Minimizing the schedule length requires maximum transfer of information such that the sum of the transmission rates of the links in S is maximized as

$$\sum_{i \in S} r_i(S) = W log_2(1 + \frac{\sum_{i \in S} P_i}{N_0}) \tag{2}$$

This maximum information transfer can be achieved by SIC using any decoding order. There are $|S|!$ possible orderings of the links in S. For each decoding order, the rates of the links are determined by eliminating the previously decoded signals from the composite signal and treating the remaining signals as interference. The capacity region achieved by SIC is larger than that achieved with TDMA.

IV. Scheduling for Delay Minimization

A. Optimal Linear Programming (LP) Formulation

The optimal transmission rate for any subset S of the link set $\mathcal{L}$ is a linear combination of the transmission rates obtained by using SIC in $|S|!$ decoding orderings of this subset. The scheduling problem can therefore be formulated as an LP problem given the transmission rates of the SIC in $|S|!$ decoding orderings for all $S \subseteq \mathcal{L}$.

Let $\mathcal{E} = \{\mathcal{E}_k : 1 \leq k \leq |\mathcal{E}|\}$ be the set of all ordered subsets of the link set $\mathcal{L}$ of maximum size K due to the decodability limit stated in Section II. Let R denote an $L \times |\mathcal{E}|$ matrix such that the element in the i-th row and k-th column of R is the transmission rate of link i when the transmission of link i is decoded in the order specified in $\mathcal{E}_k$ by using SIC if link i is within $\mathcal{E}_k$ and 0 otherwise.

The optimal minimum length scheduling problem is formulated as an LP problem as

$$\min_{t} \quad \mathbf{1}^T t \tag{3a}$$

$$\text{s.t.} \quad Rt \geq F\,, \tag{3b}$$

$$t \geq \mathbf{0}\,, \tag{3c}$$

where $\mathbf{1}$ is $|\mathcal{E}| \times 1$ all-ones vector, $\mathbf{0}$ is $|\mathcal{E}| \times 1$ all-zeros vector. The variable of the LP problem is the $|\mathcal{E}| \times 1$ vector $t = [t^{(1)}, t^{(2)}, ..., t^{(|\mathcal{E}|)}]^T$, where $t^{(k)}$ is the time slot length allocated to the ordered subset $\mathcal{E}_k$. Equation (3a) represents the objective of minimum schedule length. Equation (3b) represents the requirement of meeting the traffic demands of the links in the set $\mathcal{L}$. Equation (3c) represents the nonnegativity requirement for the slot lengths. This problem looks very similar to the LP problems previously formulated for the minimum length scheduling in different types of networks [4], [8]. The main difference is in the construction of the set $\mathcal{E}$ and corresponding matrix R due to the inclusion of the ordering in building the subsets and calculation of the transmission rates, and the maximum size constraint on the subsets as a result of the decodability limit.

The complexity of the solution based on this LP formulation mainly comes from the large number of variables in the LP problem. Since $\mathcal{E}$ contains all ordered subsets of the link set $\mathcal{L}$ of maximum size K, the number of variables in the LP problem is $O(L^K)$. The worst case complexity of the LP problem is then $O(L^{4K+1}d)$, where d is the maximum number of bits to record one parameter among all parameters of the problem, using Interior Point Method [11]. On the other hand, the optimal solution of this LP problem contains at most L non-zero variables based on Caratheodory theorem. This suggests the use of the CGM where the most useful columns of R are generated iteratively as needed.

B. Column Generation Method (CGM) based Heuristic Algorithm

CGM decomposes the large scale original LP problem into two subproblems, a Restricted Master Problem and a Pricing Problem, and solves them iteratively.

1) Restricted Master Problem (RMP): The RMP is similar to the original problem except that only a small subset of the feasible ordered subsets of $\mathcal{L}$ denoted by $\mathcal{E}^p$ and corresponding transmission rate matrix R^p is used instead of $\mathcal{E}$ and R respectively. $\mathcal{E}^p$ and R^p matrix can be chosen arbitrarily in the first iteration of the CGM as long as the feasibility of the LP problem is guaranteed. An example feasible initial set is $\mathcal{E}^p = \{(i, i+1, ..., i+K-1) | 1 \leq i \leq L-K+1\}$. The R^p matrix is then extended by including a column that improves the objective function in the following iterations until no such column can be found. To determine which column to include, we define the reduced cost of a column, which leads to the formulation of the PP, next.

The reduced cost of a column $r^{(k)}$ in the matrix R is denoted by $c^{(k)}$ and given by

$$c^{(k)} = 1 - \left(t^d\right)^T r^{(k)} \quad (4)$$

where t^d is the dual optimal solution of the RMP. The PP aims to find the column with minimum $c^{(k)}$ value among all columns in the R matrix. If this minimum $c^{(k)}$ value is greater than 0 then the CGM algorithm stops providing the optimal solution to the original problem. Otherwise, CGM continues by including the column with minimum $c^{(k)}$ value in the matrix R^p [7].

2) Pricing Problem (PP): The PP aims to find the ordered subset $(l_1, l_2, ..., l_K)$ that minimizes the reduced cost as formulated in Equation (4) so maximizes the price defined as

$$p_{(l_1,l_2,...,l_K)} = \sum_{u=1}^{K} t^d_{l_u} W log_2 (1 + \frac{P_{l_u}}{N_0 + \sum_{v=u+1}^{K} P_{l_v}}) \quad (5)$$

where t^d_l is the l-th element of the vector t^d representing the dual optimal solution of the RMP in the current iteration. The PP is harder than the problem that aims to find the subset that maximizes the price without considering the ordering, which is a Non-linear Integer Programming Problem [8]. Solving the PP therefore requires exponential effort [12]. We propose the following heuristic algorithm to efficiently and rapidly solve the PP.

3) Heuristic Algorithm for PP: The heuristic algorithm is based on iteratively including the links one at a time such that the price of the resulting ordered set is maximized. The algorithm starts with the empty set. In each iteration, the set is extended with the link that maximizes the price when included as the last element of the set. The algorithm stops when the set contains K links or the addition of a link does not increase the price of the set. This heuristic algorithm is very similar to that proposed in [8] except the inclusion of the ordering in the calculation of the price of a set as formulated in Equation (5).

4) Complexity of CGM based Heuristic Algorithm: Let Z be the number of iterations in the CGM algorithm. In each iteration of the CGM, the RMP solves an LP problem of at most $L + Z$ variables resulting in the worst case complexity of $O(L(L + Z)^4 d)$ using Interior Point Method [11]. The heuristic algorithm for PP on the other hand has complexity of $O(LK)$. Therefore, the overall complexity of the CGM based heuristic algorithm is $O(ZL(L+Z)^4 d)$. Since the complexity of Z is $O(L^K)$ in the worst case, the worst case complexity of the CGM based heuristic algorithm is worse than that of the optimal LP solution as expected [7]. However, its average runtime is much better than that of the optimal LP solution due to the average Z value much lower than $O(L^K)$ as demonstrated in Section V.

V. Simulation Results

The goal of this section is to compare the performance of the proposed CGM based heuristic algorithm with that of the optimal LP solution, the TDMA algorithm and the only previously proposed heuristic algorithm in the literature [6], which is based on scheduling any K links from the link set $\mathcal{L}$ in an arbitrary order at any time until all the links have satisfied their traffic demands. We denote the optimal LP formulation, the TDMA algorithm, the proposed CGM based heuristic algorithm and the previously proposed greedy heuristic algorithm by "OPT", "TDMA", "CGM-HS" and "GHS" respectively.

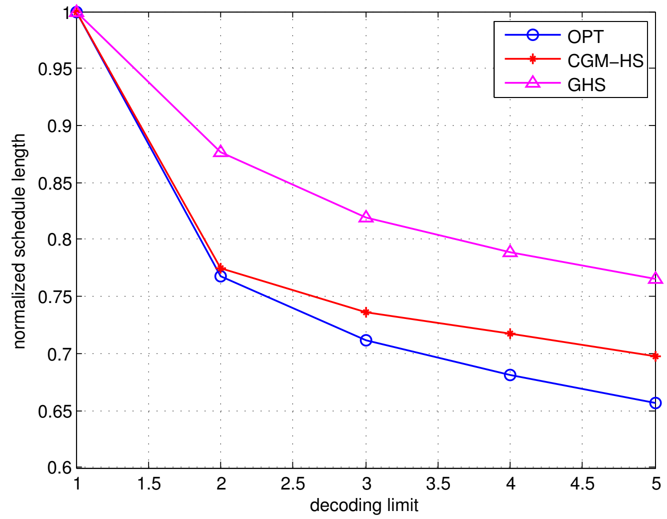

Fig. 1. Normalized schedule length for different decoding limits in a 20-link network.

We used MATLAB on a computer with a 2.5 GHz CPU and 4 GB RAM to run the simulations. Simulation results are obtained by averaging over 1000 independent random network topologies where the nodes are randomly distributed within a square of length 100m and the receiver is placed in the middle of the square. The traffic demands of the links are assigned randomly between 1 and 10 Mbits. The attenuation of the links is determined by using Rayleigh fading with scale parameter Ω set to the mean power level determined by using the path loss and shadowing model given by $PL(d) = PL(d_0) - 10\gamma \log_{10}\left(\frac{d}{d_0}\right) + Z$, where d is the distance between the transmitter and receiver, d_0 is the reference distance, $PL(d)$ is the path loss at distance d, γ is the path loss exponent and Z is a Gaussian random variable with zero mean and σ_z^2 variance. The parameters of the model used in the simulations are $\gamma = 3$, $\sigma_z^2 = 2$, $PL(d_0) = 1dB$, $d_0 = 1m$, $N_0 = 10^{-5}W/Hz$, $W = 1$MHz, $p_{max} = 1W$.

Figs. 1 and 2 show the schedule length normalized by that of the OPT solution at $K = 1$ and the average runtime respectively for OPT, CGM-HS and GHS algorithms with different decoding limits in a 20-link network. The TDMA corresponds to the decoding limit of 1. We observe that the decoding limit of even 2 decreases the schedule length of the TDMA by 25%. The schedule length then further decreases as the decoding capability of the receiver increases but with diminishing improvements. CGM-HS performs very close to the OPT and considerably better than the GHS. On the other hand, the average runtime of the CGM-HS is stable as the decoding capability increases compared to the exponential increase of that of the OPT. Although the average runtime of the CGM-HS is worse than that of the OPT at small values of the decoding limit, increasing the decoding limit results in an exponential increase in the number of the ordered link

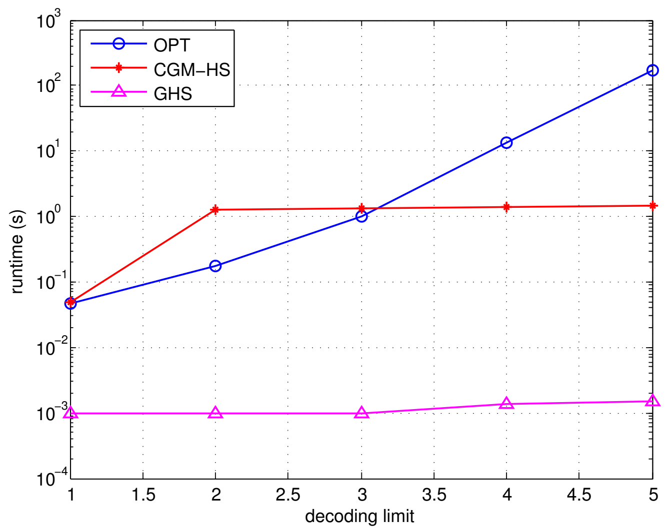

Fig. 2. Average runtime for different decoding limits in a 20-link network.

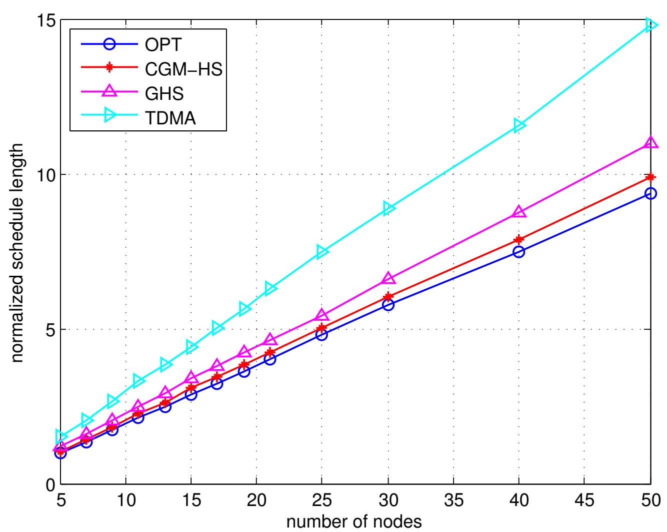

Fig. 3. Normalized schedule length with the decoding limit $K = 5$ for different number of nodes.

subsets making the solution of the OPT intractable for large values of the decoding limit.

Figs. 3 and 4 show the schedule length normalized by that of the OPT solution in a 5-link network and the average runtime respectively for OPT, TDMA, CGM-HS and GHS algorithms with the decoding limit $K = 5$ for different number of nodes. We observe that OPT, CGM-HS and GHS decreases the schedule length of TDMA considerably. Moreover, CGM-HS again performs much closer to the OPT than the GHS. The runtime of the CGM-HS on the other hand is robust to the increasing number of the links.

VI. Conclusion

In this paper, we propose a heuristic algorithm based on the CGM to solve the optimal scheduling problem to minimize the schedule length given the traffic demands of the links in SIC based single-hop multiple access networks. Simulations show that this algorithm performs very close to the optimal

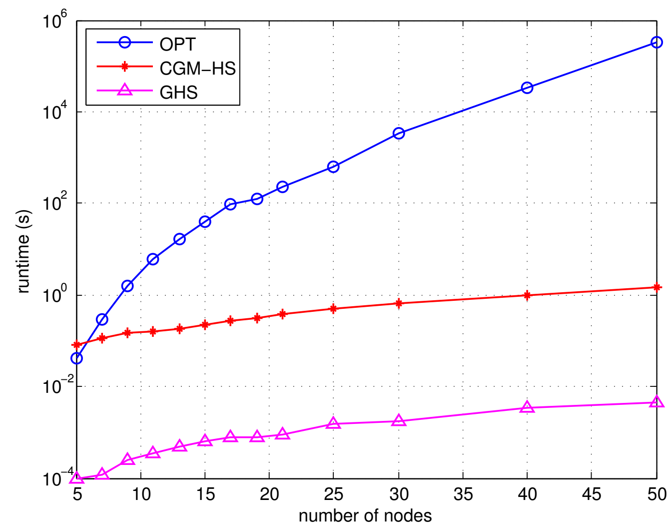

Fig. 4. Average runtime with the decoding limit $K = 5$ for different number of nodes.

formulation with runtime robust to the increasing number of the links and decoding capability of the receiver and much smaller than that of the optimal algorithm.

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