Optimal Placement of Relay Nodes for Energy Efficiency in Sensor Networks *

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February 21, 2006

Abstract

Energy efficient system design in wireless sensor networks has been previously discussed at different levels of the network protocol stack so as to provide the maximum possible lifetime of a given network. This paper proposes a novel idea to save energy through extra relay nodes by eliminating geometric deficiencies of the given topology. Given the sensing locations, the problem is to determine the optimal locations of relay nodes together with the optimal energy provided to them so that the network is alive during the desired lifetime with minimum total energy. We first formulate the problem as a nonlinear programming problem. We then propose an approximation algorithm based on restricting the locations where the relay nodes are allowed to a square lattice. This algorithm approximates the original problem with performance ratio of as low as 2 by trading complexity. For the parking lot application we consider, the relay nodes provide a significant decrease in the total energy required to achieve a specific lifetime.

1 Introduction

A wireless sensor network consists of a group of nodes, each comprising one or more sensors, a processor, a radio and a battery. Such sensor networks are expected to find widespread use in such applications as traffic monitoring on freeways or urban street intersections, seismic and medical data-gathering because of their low cost, small size and wireless data transfer [12].

Research studies conducted on wireless sensor networks fall into one of two categories: sensor placement and energy management. The objective of energy management is to increase network operational lifetime since the nodes in a sensor network may not be charged once their energy is drained. Several energy conserving protocols have been proposed at Medium Access Control (MAC) layer [7, 14], routing layer [13, 2, 6] and application layer [11, 4] of the network protocol stack.

The common goal in sensor placement research on the other hand is to determine the location of the sensor nodes that minimizes the cost while providing high coverage and resilience to failures [1, 5]. High coverage of the areas that the sensor nodes are expected to sense however may bring some geometric deficiencies so limiting energy provisioning to the existing sensor nodes may not yield the most efficient solution. An example system is illustrated in Figure 1. Figure 1(a) illustrates the placement of the sensor nodes in a parking lot. Each sensor node corresponds to a parking space. The resulting topology

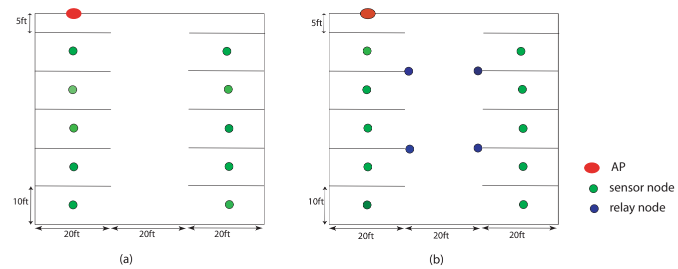

Figure 1: a) Locations of the AP and sensor nodes in a parking lot. b) Possible locations for the relay nodes.

however is not uniform due to the large distance between two sides of the parking lot so extra relay node placement between these sides as shown in Figure 1(b) may decrease energy consumption. The placement of the relay nodes and the energy saving through them is the main idea of this paper.

Relay node placement can be interpreted as an additional step between sensor placement and energy management based on the connectivity of the network resulting from the placement of the nodes. We assume that the locations and sampling rates of the sensor nodes are predetermined by some sensor placement algorithm in order to make our algorithm suitable for any application and to avoid complexity in the first step of the study. One executes our algorithm to determine the optimal locations of relay nodes together with the optimal energy provided to them so that the network is alive during the desired lifetime with the minimum total energy. The usage of energy management protocols then provides the desired lifetime.

To the best of our knowledge, the use of relay nodes is only introduced in [3] and [10]. [3] aims at placing minimum number of relay nodes to maintain the connectivity of a sensor network with a limited transmission range. The problem is formulated as a Steiner Minimum Tree with minimum number of Steiner points (SMT-MSP) problem and then approximated with performance ratio of 3. However, only decreasing the transmission range without taking into account the energy spent in the circuit and along multiple hops may not be always energy efficient. Rather than providing connectivity [1, 0] considers maximizing the network lifetime by allocating a total amount of additional energy among sensor nodes and relay nodes. Although the energy provided to the nodes is continuous, this problem is formulated as a mixed-integer non-linear programming problem due to the constraint of maximum number of nodes that can be assigned additional energy. Moreover, the proposed heuristic algorithm cannot be compared to the optimal solution.

Another idea that is related to the relay node placement is to deploy

*Research supported by NSF EIS-042445 grant.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2006 proceedings.

a lot of sensor nodes and turn only a subset of the nodes on at any given time to save energy [8]. However, putting a lot of sensor nodes inside the interested coverage area may be very costly. Moreover, it is important to understand the best placement of the nodes over a larger area that even includes the locations that do not need to be sensed.

This paper is organized as follows: Section 2 represents the framework for formulating the problem. The problem of minimizing the total cost for a network allowed to contain relay nodes only in predetermined locations is formulated as linear programming and mixed-integer linear programming problems for continuous and discrete energy allocation respectively in Section 3. In Section 4, the general problem that removes the location restriction of the relay nodes is formulated as a non-linear programming problem and then approximated by a constant factor algorithm where the constant can be decreased to any value as low as 2 by increasing the complexity of the problem. Simulation results for placing the relay nodes in a parking lot are in Section 5. Section 6 concludes the paper.

2 System Model

Consider a wireless ad hoc network that consists of one access point (AP), several sensor nodes that generate data for transfer to the AP and several relay nodes. Sensor nodes can act as both source and router whereas relay nodes only act as router. Sensor and relay nodes are static once they are deployed.

The topology information of the sensor network is represented by a graph $G = (V, E)$, in which $V = \{1\} \cup V_s \cup V_r$ is the set of nodes including AP, node 1, sensor nodes, $V_s = [2, N]$, and relay nodes, $V_r = [N+1, M]$. $(i, j) \in E$ if nodes i and j are in the transmission range of each other.

Sensor nodes in the network are assumed to generate data at a specific rate, g_i packets per unit time at node $i, i \in [1, N]$. These rates are estimated at the beginning of the deployment based on the application and the location of the sensors.

The power consuming parts in a sensor node are radio, sensor and microprocessor. The energy spent in sensor represents the energy consumption constant over time, i.e. that does not depend on communication protocol the network is using. In the formulation, the energy spent per unit time in sensing is denoted by $p_s g_i$, where p_s is the energy spent in obtaining the samples in one packet.

The energy spent in radio, on the other hand, completely depends on the communication between the nodes. In the formulation, the energy spent in transmission and reception are represented by $\Sigma_j p_{tx,ij} f_{ij}$ and $\Sigma_j p_{rx} f_{ji}$ respectively, where $p_{tx,ij}$ is the energy spent for the transmission of a packet from node i to node j in unit time, p_{rx} is the energy spent for the reception of a packet in unit time and f_{ij} is the average time spent for the reception of packets at node j from node i in unit time. $p_{tx,ij}$ is given by $p_{tx}(d_{ij})$ in which d_{ij} is the distance between nodes i and j and the function p_{tx} that maps distance to the transmission power depends on the environment, the operating frequency and the encoding. p_{tx} is assumed to be monotonically non-decreasing function of the distance between the transmitter and receiver. Without loss of generality, we assume that MAC protocol is successful in putting the radio of the nodes in sleep mode if they are not the transmitter or receiver of a packet. The power consumed by the microprocessor and by the radio in sleep mode are assumed to be negligible.

The operational lifetime of the sensor network is defined to be the maximum time duration during which all nodes in the network are alive, i.e. the time until the first node dies, since sensor network monitoring can be impaired by the early death of some nodes and the possible disconnectedness of some other nodes as a result.

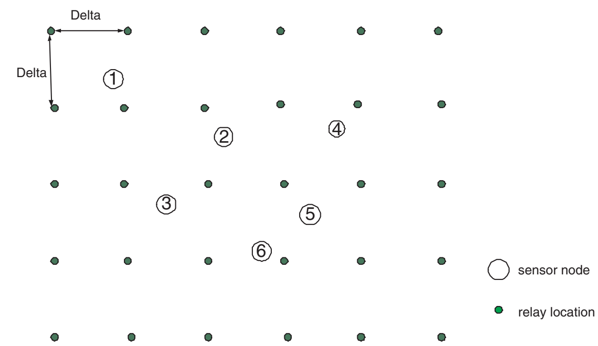

Figure 2: Example scenario for placing relay nodes in predetermined locations.

3 Relay Nodes in Predetermined Locations

In this section, we determine optimal energy distribution for sensor and relay nodes to achieve the desired lifetime for the case in which the number of possible locations allowed for the relay nodes is finite. Consider the example scenario illustrated in Figure 2. The whole area where the nodes are distributed is divided into grids. At first, we are given the location of the sensor nodes. Then the grid points shown in the figure are given as a set of possible locations for the relay nodes. We would like to assign the energy to each of these grid points and sensor nodes while minimizing their total to achieve the objective lifetime.

Recall that the topology information of the sensor network is represented by a graph $G = (V, E)$ where each edge $(i, j) \in E$ is associated with a transmission power $p_{tx,ij}$. Since the distance between all the nodes in V is predetermined, the transmission power is calculated beforehand by $p_{tx,ij} = p_{tx}(d(i, j))$ for each $(i, j) \in E$.

Figure 3 formulates the problem of minimizing the total energy provided to the nodes for an objective lifetime t_d.

Minimize $\sum_{i=1}^{M} e_i$
Subject to: $f_{ij} >= 0$ for $i, j \in [1, M]$
$e_i >= 0$ for $i \in [1, M]$
$\Sigma_j f_{ij} - \Sigma_j f_{ji} = g_i \, tt$ for $i \in [2, N]$
$t_d(\Sigma_j p_{tx,ij} f_{ij} + \Sigma_j p_{rx} f_{ji} + p_s g_i) <= e_i$ for $i \in [2, N]$
$\Sigma_j f_{ij} - \Sigma_j f_{ji} = 0$ for $i \in [N+1, M]$
$t_d(\Sigma_j p_{tx,ij} f_{ij} + \Sigma_j p_{rx} f_{ji}) <= e_i$ for $i \in [N+1, M]$

Figure 3: Optimization problem to achieve a desired lifetime t_d with finite number of allowed locations for relay nodes.

The variables of the problem are the packet flow rates f_{ij}, which is the average time spent for the reception of the packets at node j from node i in unit time, and the battery energy e_i for $i \in [1, M]$. The goal

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of the optimization problem is to minimize $\sum_{i=1}^{M} e_i$.
The first and second constraints represent the non-negativity constraint of flows and energy respectively.
The third and fifth constraints represent the requirement of the net flow out of each sensor node and relay node respectively. The expected net flow out of each sensor node $i \in [2, N]$ should be equal to the time required to transmit the packets generated in that node per unit time, $g_i tt$, where tt is the transmission time of one packet, while the net flow out of each relay node location, which corresponds to the nodes $i \in [N+1, M]$, should be zero.
The fourth and sixth constraints guarantee that all the nodes are alive during the desired lifetime t_d. The term in parentheses refers to the energy spent per unit time. It includes the energy spent in transmission and reception of packets for both sensor and relay nodes with the additional energy spent in sensing for sensor nodes.
If the optimization variables $f_{ij}, i, j \in [1, M]$ and $e_i, i \in [1, M]$ are allowed to be any real non-negative number, the problem is a linear programming (LP) problem, which can be solved by simplex method or other polynomial complexity methods.
The formulation is transformed into a mixed-integer linear programming problem for the case where the manufacturer produces a standard package with a standard amount of battery energy for the nodes. Let e_c denote the energy of the standard battery package. Then the optimization variable e_i is replaced by $x_i e_c$ for $i \in [1, M]$ in which x_i is the integer optimization variable. The goal is to minimize the total number of batteries that should be provided, which is given by $\sum_{i=1}^{M} x_i$.

4 Relay Nodes in Any Location

This section extends the results of the previous section by removing the restriction of predetermined locations for the relay nodes.
The topology information of the sensor network is represented by $G = (V, E)$. The fixed locations of the AP and sensor nodes are given by $L_s = \{l_1, l_2, ..., l_N\}$ whereas the variable locations of the relay nodes are given by $L_r = \{l_{N+1}, ..., l_M\}$. Notice that the transmission power $p_{tx,ij}$ associated with a link $(i, j) \in E$ is variable if either i or j, or both are relay nodes. This general problem is formulated as a non-linear programming problem in Figure 4.

Minimize $\sum_{i=1}^{M} e_i$
Subject to: $f_{ij} >= 0$ for $i, j \in [1, M]$
$e_i >= 0$ for $i \in [1, M]$
$\Sigma_j f_{ij} - \Sigma_j f_{ji} = g_i\, tt$ for $i \in [2, N]$
$t_d(\Sigma_j p_{tx,ij} f_{ij} + \Sigma_j p_{rx} f_{ji} + p_s g_i) <= e_i$ for $i \in [2, N]$
$\Sigma_j f_{ij} - \Sigma_j f_{ji} = 0$ for $i \in [N+1, M]$
$t_d(\Sigma_j p_{tx,ij} f_{ij} + \Sigma_j p_{rx} f_{ji}) <= e_i$ for $i \in [N+1, M]$
$p_{tx,ij} = p_{tx}(d(i,j))$ for $i, j \in [1, M]$
$d(i,j)^2 = |l_i - l_j|^2$ for $i, j \in [1, M]$

Figure 4: Optimization problem to achieve a desired lifetime t_d without any restriction on the locations of relay nodes.

The variables of the problem are the packet flow rates f_{ij}, $i, j \in [1, M]$, the battery energy e_i, $i \in [1, M]$, and the location of the relay nodes $l_i, i \in [N+1, M]$. The goal of the optimization problem is to minimize $\sum_{i=1}^{M} e_i$.
The first six constraints are the same as those in Figure 3. The last two constraints are added since the distance between nodes i and j where either $i \in V_r$ or $j \in V_r$ is variable, resulting in variable $p_{tx,ij}$.
The formulation is transformed into a mixed-integer nonlinear programming problem for the discrete energy assignment scenario. The optimization variable e_i is replaced by $x_i e_c$ for $i \in [1, M]$ in which x_i is the integer optimization variable. The goal is to minimize the total number of batteries that should be provided, which is given by $\sum_{i=1}^{M} x_i$.
This problem is not a convex optimization problem. We therefore try to find a constant factor approximation algorithm. We basically restrict the allowable locations for the relay nodes and compare the total energy allocation to the optimal one. The complexity of the resulting polynomial algorithm is expected to increase as its approximation constant decreases.
We consider a grid structure for the allowable locations of relay nodes. Assume that the nodes are allowed to be distributed inside a rectangle of area A. The number of possible locations for the relay nodes on the square lattice in which the distance between two neighboring lattice vertices is Δ is given by $\frac{A}{\Delta^2}$. This means that as the distance Δ decreases, the number of variables in the formulation given in Figure 3 increases by a factor of $\frac{1}{\Delta^2}$. In the following, we find the dependence of the approximation constant on this distance Δ so as to understand the cost of restricting allowable locations for the relay nodes.
Theorem 1 *Let e_i be continuous and $G \subset R^2$ be the set of all vertices of a square lattice in which the distance between two neighboring lattice vertices is Δ. Then the optimal total energy required for the case where the relay nodes are only allowed to be on the vertices of G is at most $2max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)}$ times the optimal total energy required for the case where there is no restriction on the location of the relay nodes, which we call* no-restriction case*.*
Proof We prove this theorem by providing an algorithm that takes the graph representing the optimal solution for the no-restriction case and obtains a solution for the grid structure by splitting each relay node in the optimal graph.
For this purpose, we first introduce some notation. Let the optimal solution of the no-restriction case be given by the graph $G_{nr} = (V_{nr}, E_{nr})$, where V_{nr} contains the sensor nodes and relay nodes that are used to carry packets and $E_{nr} = \{(i,j) | f_{ij}^{opt} > 0, i, j \in V_{nr}\}$, where f_{ij}^{opt} refers to the optimal flow rate from node i to node j. Let us initialize the graph that will represent a solution of the grid structure with $G_g = (V_g, E_g)$, where $V_g = V_s$ is the set of sensor nodes and $E_g = (V_s \times V_s) \cap E_{nr}$ contains only the edges of E_{nr} between the sensor nodes.
The nodes corresponding to a relay node i are defined to be the vertices of the lattice node i is in. If node i is located on a vertex of the lattice, then there is only one corresponding node. Otherwise, there are 4 corresponding nodes. The node corresponding to a sensor node is itself.
The excess at the node k corresponding to a relay node i is defined as follows:

$$e_k^i = \Sigma_j I(i,j,k) f_{ij}^{opt} - \Sigma_j I(i,j,k) f_{ji}^{opt} \quad (1)$$

where $I(i,j,k) = 1$ if k is the node corresponding to node i that is closest to one of the corresponding nodes of node j and $I(i,j,k) = 0$ otherwise.

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The pseudocode of the transformation algorithm is given in Figure 5.

For each relay node $i \in V_{nr}$
  add one relay node at each of the vertices corresponding to node i to V_g if it is not already in V_g
  if $(i,j) \in E_{nr}((j,i) \in E_{nr})$ and node j is a sensor node or relay node that has already been processed
    add a directed arc between the closest nodes corresponding to node i and j to E_g with the corresponding flow rate $f_{ij}^{opt}(f_{ji}^{opt})$
For each relay node $i \in V_{nr}$
  determine the excess e_k^i at each corresponding relay node $k \in V_g$
  solve the resulting transportation problem by assigning flows along the shortest path between these relay nodes
  update G_g with the resulting flows from the transportation problem

Figure 5: Algorithm to transform optimal solution for the no-restriction case to a solution for the grid structure

In the first part of the algorithm, the relay nodes in G_{nr} are split so that all the relay nodes in the resulting graph G_g are on the vertices of the lattice. Each link $(i,j) \in E_{nr}$ is transformed into a link between two nodes, one corresponding to node i and the other corresponding to node j in G_g. Although this transformation does not change the flows f_{ij} incident to sensor nodes, it affects the flow balance equations at the relay nodes. The second part of the algorithm aims at satisfying the flow balance equations in G_g by adding flows between the vertices corresponding to relay nodes in G_{nr}. The sum of e_k^i over all the corresponding vertices k of node i should be zero. Flows are assigned between these vertices to make the resulting e_k^i equal to zero for all i. The essence of the algorithm lies in the fact that the resulting G_g keeps the flow rates incident on sensor nodes the same while possibly increasing the number of relay nodes between them compared to G_{nr} due to the restriction of placing them at the vertices of the lattice. The first 2 constraints in LP problem in Figure 3 are obviously satisfied for G_g resulting from the above transformation. If the flow balance equations for the nodes are satisfied for G_{nr}, so do they for G_g.
The flows incident on sensor nodes do not change. Therefore, the reception energy at the sensor nodes is the same. If the sensor node i and the relay node j are in the same lattice, then the maximum distance between the sensor node and the closest corresponding relay node is $\frac{1}{\sqrt{2}}\Delta$. The maximum ratio of the energy consumption to the optimum energy consumption is $\frac{p_{tx}(\frac{1}{\sqrt{2}}\Delta)}{p_{tx}(d_{ij})}$. The maximum value of this ratio is given by $\frac{p_{tx}(\frac{1}{\sqrt{2}}\Delta)}{min_{d_{ij},i,j\in V_{nr}} p_{tx}(d_{ij})}$, which is upper bounded by $\frac{p_{tx}(\frac{1}{\sqrt{2}}\Delta)}{min_d p_{tx}(d)}$, which is more than or equal to 1. On the other hand, if they are on different lattices, then the distance between the corresponding relay node and the sensor node may decrease or increase from d_{ij} to at most $\sqrt{d_{ij}^2+\Delta^2}$. This increase corresponds to transmission energy increase of a factor of $\frac{p_{tx}(\sqrt{d_{ij}^2+\Delta^2})}{p_{tx}(d_{ij})}$, which is upper bounded by $max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)}$, which is more than or equal to 1. The total resulting energy consumption at the sensor nodes therefore increases by at most a factor of $max(\frac{p_{tx}(\frac{1}{\sqrt{2}}\Delta)}{min_d p_{tx}(d)}, max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)}) = max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)}$ since $min_d p_{tx}(d) = p_{tx}(0)$.
The total flow rate incident on relay nodes is at most doubled to balance flow equations in second part of the algorithm so the reception energy of the relay nodes is at most doubled.
The energy spent for the transmission of the flow f_{ij} from relay node i to node j is $p_{tx}(d_{ij})f_{ij}$ in G_{nr}. During the first step of the transformation algorithm, if the nodes are in the same lattice and node j is a relay node, the closest corresponding relay nodes are the same so no energy is required to transmit the flow. If the nodes are in the same lattice and node j is a sensor node, the maximum distance between the sensor node and corresponding relay node is $\frac{1}{\sqrt{2}}\Delta$ resulting in energy consumption increase by at most a factor of $\frac{p_{tx}(\frac{1}{\sqrt{2}}\Delta)}{min_d p_{tx}(d)}$. If they are on different lattices and node j is a relay node, the transmission energy from the relay node corresponding to node i to that corresponding to node j is less than $p_{tx}(d_{ij})f_{ij}$ since they are closer than d_{ij}. If they are on different lattices and node j is a sensor node, the transmission energy increases by at most a factor of $max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)}$. The maximum ratio of the energy consumption at this stage to the optimum energy consumption is upper bounded by $max(\frac{p_{tx}(\frac{1}{\sqrt{2}}\Delta)}{min_d p_{tx}(d)}, max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)}) = max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)}$, which is more than or equal to 1.
During the second step, additional flows are added to satisfy the flow balance equations. The maximum additional flow between the relay nodes at the corresponding vertices for f_{ij} is f_{ij} transmitted at maximum distance $\sqrt{2}\Delta$ resulting in transmission energy $p_{tx}(\sqrt{2}\Delta)f_{ij}$. The ratio of this energy to the optimum energy consumption is upper bounded by $\frac{p_{tx}(\sqrt{2}\Delta)}{min_d p_{tx}(d)}$. The total transmission energy at the relay nodes therefore increases by at most $\frac{p_{tx}(\sqrt{2}\Delta)}{min_d p_{tx}(d)} + max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)}$, which is upper bounded by the following:

$$2max(\frac{p_{tx}(\sqrt{2}\Delta)}{min_d p_{tx}(d)}, max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)}) \quad (2)$$

$$\leq 2max(\frac{p_{tx}(\sqrt{2}\Delta)}{min_d p_{tx}(d)}, max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)}) \quad (3)$$

$$\leq 2max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)} \quad (4)$$

The overall energy consumption increases by at most a factor of $2max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)}$. Since the optimal energy required for the relay nodes restricted to be placed at the vertices of the lattice is less than the energy required for G_g, the approximation constant is $2max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)}$.

□

Theorem 2 *Let e_i be discrete, i.e. $e_i = x_i e_c$ in which e_c is the battery energy and x_i is integer for each $i \in [2, M]$, and $G \subset R^2$ be the set of all vertices of a square lattice in which the distance between two neighboring lattice vertices is Δ. Then the optimal total energy required for the case where the relay nodes are only allowed to be on the vertices of G is at most $4\lceil max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)}\rceil$ times the optimal total energy required for the case where there is no restriction on the location of the relay nodes, which we call* no-restriction case.

Proof The proof is again based on an algorithm that transforms the graph representing the optimal solution for the no-restriction case to a

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solution for the grid structure as in the proof of Theorem 1. The additional restriction here is that we have to look at each node separately due to the discrete energy allocation requirement.

As a result of the transformation algorithm explained in the proof of Theorem 1, the total energy consumption at the sensor nodes increases by at most a factor of $\lceil max_d \frac{p_{tx}(\sqrt{d^2+\Delta^2})}{p_{tx}(d)} \rceil$.

The flow rate incident on each corresponding relay node so the reception energy of the corresponding relay node is less than or equal to that of the relay node itself.

The energy spent for the transmission of the flow f_{ij} from node i to node j is $p_{tx}(d_{ij})f_{ij}$ in G_{nr} whereas the maximum energy spent in transmission at each corresponding vertex of the relay node is the maximum of $p_{tx}(\sqrt{2}\Delta)f_{ij}$ and $p_{tx}(\sqrt{d_{ij}^2+\Delta^2})f_{ij}$. The ratio of the latter to the first is at most $max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)}$. The resulting energy increase ratio at each relay node is therefore $\lceil max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)} \rceil$. Since there are at most 4 relay nodes in G corresponding to each relay node in G_{nr}, the total energy consumption increases by at most a factor of $4\lceil max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)} \rceil$.

Since the optimal energy required for the relay nodes restricted to be placed at the vertices of the lattice is less than the energy required for G_g, the approximation constant is $4\lceil max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)} \rceil$.

□

Remark Note that the approximation constant can be made arbitrarily close to 2 in Theorem 1 by choosing small enough Δ.

5 Simulation

The purpose of our simulation is to examine the effect of relay nodes on the total energy provided to the network for a parking lot application.

In the simulations, the locations of the AP and sensor nodes in the parking lot are as shown in Figure 1(a). The relay nodes are placed onto the vertices of a grid in the parking lot. Figure 1(b) shows the relay nodes placed on the vertices of a square lattice of side length $20ft$. Two scenarios are simulated: constant transmission energy and variable transmission energy. In the constant transmission energy scenario, a constant transmission power is used across the network. In the variable transmission energy scenario on the other hand the nodes can adjust their transmission power according to the distance.

Figure 6 shows the correct reception probability as a function of the distance between two mica2dot nodes [12]. The quality of the link exhibits high variation when the received signal strength is below a certain level. To provide a system that is guaranteed to give a specific lifetime unless the nodes fail and a robust communication between the deployed nodes with a correct reception probability close to 1 in the constant transmission energy case, we first assume a *basic reception model*: The probability of communication is 1 if the distance between the nodes is less than a distance d and 0 otherwise, which ignores the links with a success probability less than 1 due to the inconsistencies in those links. We then extend this transmission range over k times d to obtain *extended reception model*: The probability of correct reception is 1 if the distance between the nodes is less than d and decreases linearly from 1 to 0 as the distance increases from d to kd. In this case, the transmission energy is given by the constant transmission power to reach the distance d, $p_{tx}(d)$, times $\frac{1}{p_c}$ in which p_c is the correct reception probability. This variability in the transmission power allows us to observe the effect of low quality links on the total energy required for the network.

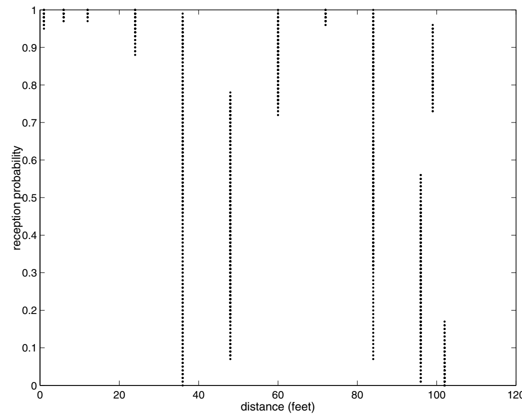

Figure 6: Correct reception probability at different distances for mica2dot motes.

The assumptions about the energy dissipation in transmit and receive modes will change the advantages of different protocols. We use the radio model described in [9]: The radio spends $E_{tr} = E_{elec} + \epsilon_{amp} d^4$ to transmit 1-bit over a transmission radius of d units whereas the radio spends $E_{rec} = E_{elec}$ to receive 1-bit, where E_{elec} is the electronics energy and ϵ_{amp} is the amplifier energy. In the figures, $E_{elec} = 100nJ/bit$ and 'ratio' denotes $\frac{\epsilon_{amp}}{E_{elec}}$. The battery package is assumed to be a pair of AA batteries, which can supply 2200 mAh at 3V. We further assume that the desired lifetime is 10 years and each sensor node consumes 1/10-th of battery energy in sampling during the desired lifetime.

Figure 7 shows the total energy consumption in the network as a function of the transmission range at 5-ft grid size for the constant transmission energy and basic reception model. If the $\frac{\epsilon_{amp}}{E_{elec}}$ ratio is large enough, there exists an optimal transmission range. The relay nodes remove the geometric deficiencies by allowing the sensor nodes decrease their transmission range. However, the transmission range should not decrease below a certain level since the increase in the number of hops to reach the AP starts to dominate the decrease in the variable part of the transmission energy, which contradicts placing minimum number of relay nodes to maintain the connectivity of a sensor network with a limited transmission range for energy efficiency [3]. Moreover the total energy consumption in the discrete energy allocation case is more than that in the continuous allocation as expected.

The effect of the grid size on the energy consumption compared to the case where no relay node is used is depicted in Figure 8. The energy saving increases as the ratio $\frac{\epsilon_{amp}}{E_{elec}}$ increases due to the dominance of the transmission energy over the circuit energy. Notice that the value of the approximation function $max_d \frac{p_{tx}(\sqrt{d^2+2\Delta^2})}{p_{tx}(d)}$ in Theorems 1 and 2 decreases as $\frac{\epsilon_{amp}}{E_{elec}}$ decreases for the same grid size Δ. Furthermore, the energy saving is observed to be less for the discrete energy allocation.

Figures 9 and 10 show the energy consumption as a function of transmission range at 5-ft grid size and grid size respectively for the constant transmission energy and extended reception model: The transmission range d means that the probability of correct reception is 1 if the distance between the nodes is less than d and decreases linearly

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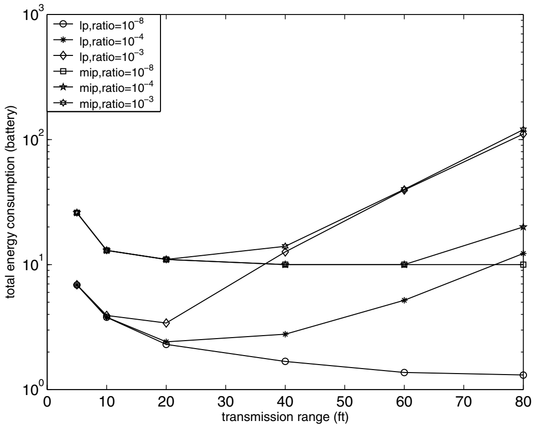

Figure 7: Energy consumption for different transmission ranges and basic reception model at 5-ft grid size.

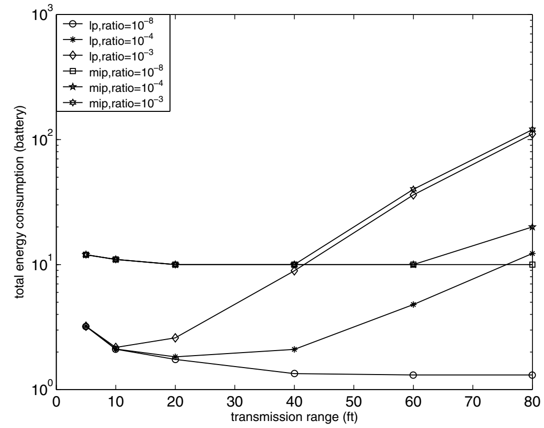

Figure 9: Energy consumption for different transmission ranges and extended reception model at 5-ft grid size.

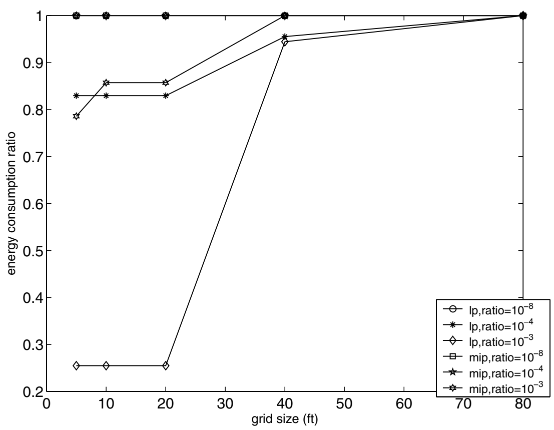

Figure 8: Energy consumption normalized by that in the 'no relay node' case for different grid sizes with basic reception model.

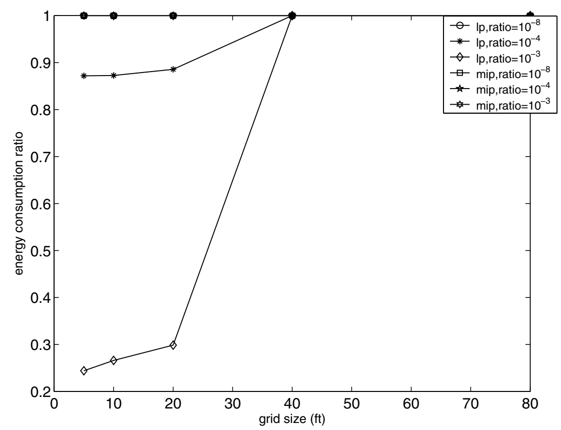

Figure 10: Energy consumption normalized by that in the 'no relay node' case for different grid sizes with extended reception model.

from 1 to 0 as the distance increases from d to $3d$. The behavior of the figures is similar to those for the basic reception model except that the energy consumption is less compared to that in the basic reception model. Moreover, the energy saving as a function of the grid size has also decreased since the transmission energy has become variable over a certain range. This of course comes at the cost of decreased robustness.

Figure 11 illustrates the energy saving by the decrease in the grid size for the variable transmission energy scenario. The energy saving is less compared to the constant transmission energy case since the nodes can save energy by adjusting their transmission power even in the 'no relay node' case. Notice that Figures 10 and 11 do not show any energy saving for the discrete energy allocation case. This is because at least one battery energy is assigned to each sensor node, which is enough to handle all the transmissions.

Figures 12 and 13 demonstrate the effect of the grid size on the energy saving for $1/10$-th of battery energy. As expected, the discrete and continuous energy allocation have similar behavior with slightly more energy saving in the continuous energy allocation.

6 Conclusion

We propose a novel idea of energy management by using relay nodes in a wireless sensor network. We assume that the locations and data generation rates of sensor nodes are predetermined by the sensor placement algorithm of the application. The problem is then to determine the optimal locations of relay nodes together with the optimal energy provided to them so that the network is alive during a desired lifetime with minimum total energy.

We formulate the problem as a nonlinear programming problem. We then propose an approximation algorithm based on restricting the locations where the relay nodes are allowed to a square lattice. The algorithm approximates the original problem with performance ratio of as low as 2 by trading complexity.

For the parking lot application we consider, the relay nodes provide a significant decrease in the total energy provided to the network by decreasing the minimum transmission range required for the network to achieve connectivity. We also observe that the transmission range should not decrease below a certain value since the increase in the constant energy dissipation in transmitter and receiver circuitry over multiple hops start to dominate the decrease in the variable energy dissipated in the transmit amplifier.

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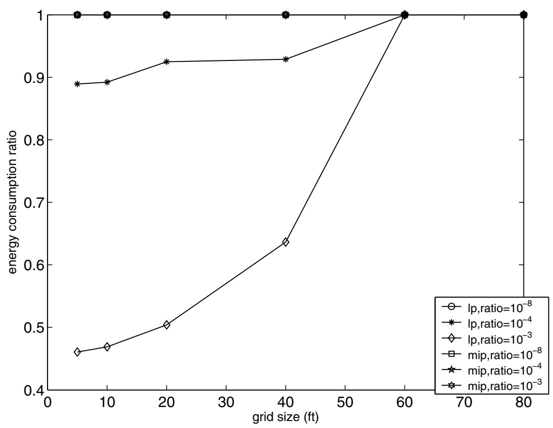

Figure 11: Energy consumption normalized by that in the 'no relay node' case for different grid sizes and variable transmission energy.

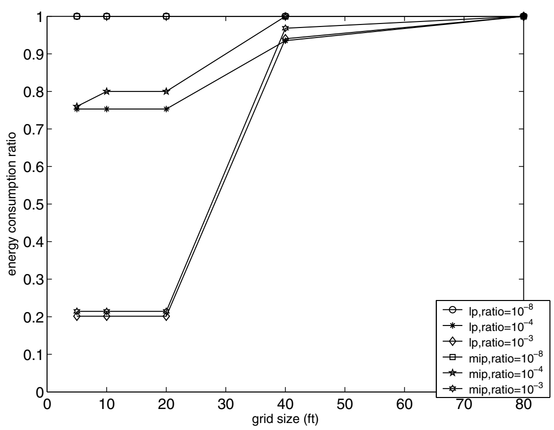

Figure 12: Energy consumption normalized by that in the 'no relay node' case for different grid sizes, basic reception model and 1/10-th battery energy.

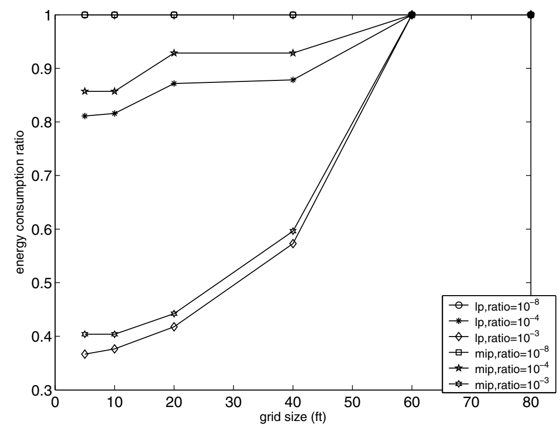

Figure 13: Energy consumption normalized by that in the 'no relay node' case for different grid sizes and variable transmission energy at 1/10-th battery energy.

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