Minimum Length Scheduling With Packet Traffic Demands in Wireless *Ad Hoc* Networks

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***Abstract*—Traditional approach to the minimum length scheduling problem ignores the packet level details of transmission protocols, meaning that a packet transmission can be divided into several data chunks each of which is transmitted at a different rate due to the difference in the set of concurrently transmitting nodes. This solution requires including packet headers for each data chunk resulting in both an increase in the system overhead and underutilization of the time slots. In this paper, we extend the previous works on minimum length scheduling by considering the transmission of the packets of arbitrary sizes in the time slots of arbitrary lengths. Given the packet traffic demands on the links, we formulate the joint optimization of the power control, rate adaptation and scheduling for minimizing the schedule length of a wireless *ad hoc* network and demonstrate the hardness of this problem. Upon solving the power control and rate adaptation problem separately, we formulate the scheduling problem as an integer programming (IP) problem where the number of variables is exponential in the number of the links. In order to solve this large-scale IP problem fast and efficiently, we propose Branch and Price Method and Column Generation Method based heuristic algorithms.**

***Index Terms*—Wireless *ad hoc* networks, packet traffic demand, power control, rate adaptation, scheduling.**

I. Introduction

SCHEDULING the wireless links and controlling their transmission power and rate in Spatial-reuse Time Division Multiple Access (STDMA) wireless networks has been investigated to minimize the detrimental effects of the interference for various objectives including maximizing total throughput [1], [2], maximizing minimum throughput [3], [4], minimizing total transmit power [5], [6] and minimizing schedule length [7]–[22]. Among these objectives, minimizing schedule length given the traffic demands of the links is increasingly used with the proliferation of the time critical applications of wireless networks such as real-time surveillance, networked control [23]. Moreover, minimum length schedule determines the traffic-carrying capability of the wireless network since minimization of the total time required for a set of links to transmit their data packets is effectively maximizing the total data transmission rate while providing fairness among the links, which are allocated enough time to transmit their packets.

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Earlier work on minimum length scheduling adopts Protocol Interference Model, which describes the interference constraints among the active links according to a conflict graph, where nodes within a certain distance can communicate as long as the receiver is separated by at least a fixed distance from any other active transmitter. Since this interference model does not allow controlling the transmission power and rate of the links due to the predetermined conflict graph, the time is partitioned into slots of fixed length allowing the transmission of a fixed length packet. The algorithms developed for minimum length scheduling using Protocol Interference Model aim to satisfy either the uniform traffic demand of the links activating each link for at least one time slot during the schedule [7], [8] or non-uniform traffic demand considering many-to-one transmission activating each link for multiple time slots during the schedule [9], [10]. Although the scheduling algorithms developed for Protocol Interference model take into account the packetized transmission allocating one packet transmission to each time slot, this model is inadequate in modeling today's radios where multiple power and rate levels are supported. Besides, this interference model does not take into account the cumulative effects of the interference due to simultaneous transmissions thus requiring the overcompensation of the interference by using a large distance of separation between the active transmitters and receivers.

Recently, *Physical Interference Model* that considers the cumulative effect of the interference in the form of Signal-to-Interference-plus-Noise Ratio (SINR) and allows including the transmission power and rate of the links as a variable in minimizing the length of the schedule has gained wider acceptance. Despite the flexibility introduced by this interference model to include variable transmission power and rate, some of the algorithms developed for this optimization problem restricts the partition of the time into slots of fixed length. The length of the time slot corresponds to the transmission time of either a fixed length packet at fixed transmission rate [12], [13] or an integer number of packets the value of which is determined as a function of the rate chosen from a finite set of possible transmission rates [14], [15]. These algorithms generate sub-optimal solutions since the fixed length time slots may be underutilized by the links. The algorithms that partition the time into slots of variable length on the other hand ignore the packet level details of the transmission protocols by allocating any amount of data to the time slots with the only goal of satisfying the total traffic demand of each node [16]–[22]. This means that in the optimal solution, a packet transmission can be

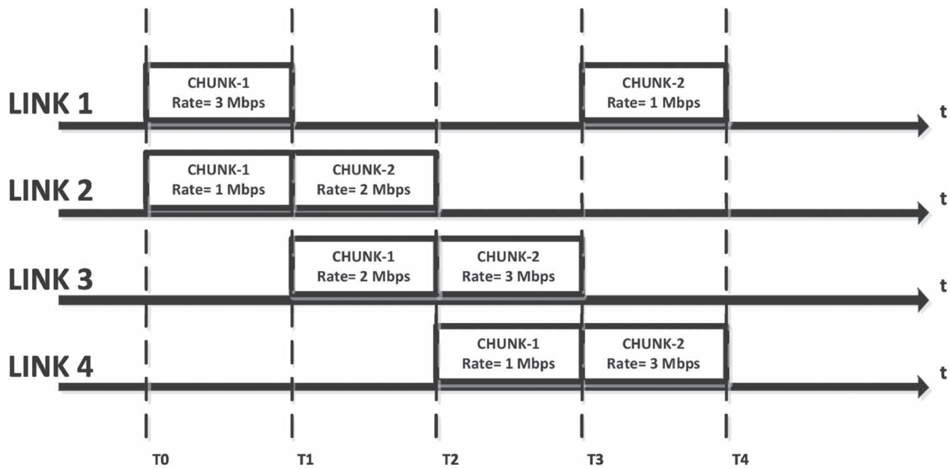

Fig. 1. Solution to the traditional formulation of minimizing the length of the schedule for the scenario where each of 4 links has one packet of 100 bits to transmit; the links are of 1 m length and uniformly distributed over an area of 15 m $\times$ 15 m; the links can only transmit at discrete rates 1, 2, 3 Mbps if the SINR value at the receiver of the link exceeds the SINR level 10, 20, 30 dB respectively [22]. Time axis values are $T0 = 0\ \mu$s, $T1 = 23.0769\ \mu$s, $T2 = 61.5384\ \mu$s, $T3 = 69.2307\ \mu$s and $T4 = 100\ \mu$s.

divided into several data chunks each of which is transmitted at different rate due to the difference in the set of concurrently transmitting nodes. Fig. 1 shows the solution to the problem of minimizing the length of the schedule formulated in [22] for a scenario where each link has one packet to transmit. Since the traffic demand in this problem is specified as the total number of bits ignoring packet level details, in the optimal solution, all the packet transmissions are transmitted in several data chunks requiring interrupting the packet transmission. For instance, packet transmission of link 1 starts at $T0$ but is interrupted at $T1$ and restarts at $T3$. Moreover, the data rate of the links is different for each data chunk due to the different set of nodes concurrently transmitting in each time interval. For instance, the data rate of link 1 is different in $T0 - T1$ and $T3 - T4$ intervals since link 2 is transmitting concurrently with link 1 in $T0 - T1$ interval whereas link 4 is transmitting concurrently in $T3 - T4$ interval. The data chunks of the same packet must therefore be transmitted as individual packets by including a packet header at the beginning in the implementation. This additional packet header increases the system overhead. Moreover, since the data rates of the links allocated to the same time slot are different, the transmission time of these headers are different for each link resulting in the underutilization of the time slots.

The goal of this paper is to determine the optimal power control, rate adaptation and scheduling with the objective of minimizing the schedule length given the *packet traffic demands* of the links and the *constraints of packet transmissions* in a wireless *ad hoc* network. We first formulate the joint optimization of the power control, rate adaptation and scheduling for minimizing the schedule length of a wireless *ad hoc* network and demonstrate the hardness of this problem. We then show that the power control and rate adaptation problem can be separated from the scheduling problem and introduce a novel problem for the optimization of power control and rate adaptation where the time required for the concurrent transmission of a set of links each having an integer number of packets is minimized. This problem determines the length of the time slot corresponding to this link set and the corresponding number of packets to be transmitted during the slot. Solving this power control and rate adaptation problem for all possible subsets of the links in the network transmitting any possible number of packets then allows formulating the problem of minimizing the schedule length as an IP problem. The resulting IP problem is large-scale with the number of variables exponential in the number of the links. We therefore incorporate elegant Branch and Price Method and Column Generation method based heuristic algorithms to solve this IP problem. Column generation method has been previously used for the minimum length scheduling problem in different contexts: fixed transmission rate and variable power [16] and variable transmission rate and variable power [19]–[22] but without considering packetized transmission; variable transmission rate and variable power but considering only a limited set of packet transmission scenarios for fixed length time slots [14], [15]. In this paper, we extend these previous works on minimum length scheduling problem by considering the transmission of the packets of arbitrary sizes in the time slots of arbitrary lengths in the optimization of power control, rate adaptation and scheduling.

The rest of the paper is organized as follows. Section II describes the system model and the assumptions used throughout the paper. In Section III, the joint optimization of power control, rate adaptation and scheduling with the objective of minimizing the schedule length is formulated considering the constraints of packet transmissions for constant, continuous and discrete rate transmission models and the solution strategy based on the decomposition of the power control and rate adaptation, and scheduling problems is described. Section IV presents the optimal power control and rate adaptation problem with the goal of determining the minimum time-slot length for the concurrent packet transmissions of a subset of the links in the network and propose both optimal and heuristic algorithms for each transmission model. Section V formulates the optimal scheduling problem as an IP problem where the number of variables is exponential in the number of the links whereas Sections VI and VII introduce Branch and Price and Column Generation Method based heuristic algorithms respectively to solve this IP problem fast and efficiently. Simulations and performance evaluation are presented in Section VIII. Finally, concluding remarks and future work are given in Section IX.

II. System Model and Assumptions

The system model and assumptions are detailed as follows:

1) The wireless *ad hoc* network consists of a set of L directed links with positive traffic demand.
2) A central controller executes the algorithm for the joint optimization of power control, rate adaptation and scheduling based on the available information on the network topology, traffic demands and channel characteristics of the links. This centralized framework is appropriate for the communication of the routers in wireless mesh networks (WMNs) that have emerged recently to partially replace the costly wired network infrastructures [24]. The changes in WMN topology is quite infrequent since the locations of the WMN routers are fixed. Moreover, the bandwidth requests of the WMN links are typically slowly varying over time and can be guaranteed with an almost static resource allocation. Besides, the centralized optimal solution provides an upper bound on the performance of any distributed algorithm since distributed algorithms rely on the local topology information in contrast to the centralized algorithms using the knowledge of the complete topology. Such an upper bound allows evaluating the performance of the distributed algorithms. Introducing distributed algorithms within the proposed framework is out of scope of this paper and subject to future work.
3) Link l has traffic demand of G_l packets of length R_l bits. Although the mathematical formulations derived in the paper allow having packets of different lengths for the same link, the formulations are easier to express with this assumption. The link traffic demands can be either given for single-hop networks or calculated by using end-to-end traffic demands in a multi-hop network with predetermined routing. The proposed framework can be extended to include routing in the optimization problem by either a two phase approach in which scheduling and routing are solved iteratively [25], [26] or a joint approach in which routing is included via flow-balance equations that map the end-to-end flow demands to the link traffic demands [3], [4], [14]. To avoid complexity in the first step of the study and better focus on the packetized scheduling, the inclusion of the routing in the optimization problem is out of scope of this paper and subject to future work.
4) We consider Time Division Multiple Access (TDMA) as MAC protocol since TDMA provides delay guarantee to the networks with predetermined topology and data generation patterns [27]. The time is partitioned into frames, which are further partitioned into slots of possibly variable lengths. Two possible interpretations can be considered for studying the problem of minimum length scheduling given traffic demands as detailed in [22]: In the first scenario, the traffic demand, i.e., G_l packets for link l, is defined as the number of packets to be transmitted in every frame. On the other hand, in the second scenario, the traffic demand is defined as the long-run average data transfer rate of G_l^s packets per second. This can then be accomplished by dividing the time axis into frames of arbitrary positive constant duration T requiring the satisfaction of $G_l = G_l^s T$ packet transmissions for every link l during each frame. Minimizing the frame length given these traffic demands is then useful for both scenarios because it permits a larger number of frames per unit time. Extending this work for the dynamic adaptation of the schedule, power and rate of the links to the arrival of packets at any instant in time is beyond the scope of this paper and subject to future work.
5) A node cannot receive and transmit simultaneously. Also, a node cannot receive from or transmit to more than one node simultaneously.
6) The transmit power can take any value below a maximum level $p_{\max}$. Although the radios are restricted to support a finite number of transmit power levels in practice, the assumption of continuous power is commonly used in most of the previous work such as [12], [16], [20]–[22] due to the simplification of the resulting problem and high approximation accuracy with the large number of discrete power levels that can be supported by the existing radios.
7) Let g_{ll} and g_{kl} be the channel gain of link l and the channel gain from the transmitter of link k to the receiver of link l respectively. We assume that the fading is slow such that the channel gain between every transmitter and receiver is fixed during the schedule. This is a common assumption used in the prior formulations of the minimum length scheduling problem in wireless *ad hoc* networks [12], [16], [17], [20]–[22]. Extending this work for fast fading wireless networks by either minimizing the mean value of the schedule length as in [19] or providing an optimal policy by employing the principles of dynamic programming while incorporating a lot of overhead for collecting the channel information at the granularity of the time slot duration as in [18] is beyond the scope of this paper and subject to future work.
8) First transmission model used in the formulations is *constant rate* transmission model in which each active link l assigned to a time slot n transmits at constant rate r if the SINR of link l is above a fixed threshold as
$$\frac{p_l^{(n)} g_{ll}}{N_0 + \sum_{k \neq l} p_k^{(n)} g_{kl}} \geq \beta_l \tag{1}$$
where $p_l^{(n)}$ is the transmit power of link l in time slot n, N_0 is the background noise and β_l is the SINR threshold to be kept by link l [12], [16].
9) Second transmission model used in the formulations is *continuous rate* transmission model in which Shannon's channel capacity formulation for an Additive White Gaussian Noise (AWGN) wireless channel is used in the calculation of the maximum achievable rate as a function of SINR as
$$x_l^{(n)} \leq W \times \log\left(1 + \frac{1}{W} \times \frac{p_l^{(n)} g_{ll}}{N_0 + \sum_{k \neq l} p_k^{(n)} g_{kl}}\right) \tag{2}$$

where $x_l^{(n)}$ is the transmission rate of link l in time slot n, W is the channel bandwidth [19]–[22], [28], [29].

10) Third transmission model used in the formulations is *discrete rate* transmission model in which a finite set of transmit rates $r = (r^1, r^2, \ldots, r^M)$ and a finite set of SINR levels $\boldsymbol{\gamma} = (\gamma^1, \gamma^2, \ldots, \gamma^M)$ are determined such that link l can transmit at rate r^c in a time slot n if the SINR achieved at the link

$$\gamma_l^{(n)} = \frac{p_l^{(n)} g_{ll}}{N_0 + \sum_{k \neq l} p_k^{(n)} g_{kl}} \tag{3}$$

is greater than or equal to γ^c from the set $\boldsymbol{\gamma}$ [17], [18]. Here $r^1 = 0$ and $\gamma^1 = -\infty$ dB so that if $\gamma_l^{(n)}$ is below the threshold γ^2, the transmission rate is 0. This is in general more realistic model than the *continuous rate* model since a wireless transmitter can only work with a limited number of rate levels as suggested by the practical realization of multiple data rates in [30].

III. Packetized Minimum Length Schedule Problem

A. Problem Formulation

The joint optimization of power control, rate adaptation and scheduling with the objective of minimizing the schedule length given the packet traffic demands and packetized transmission constraints is mathematically formulated for constant rate, continuous rate and discrete rate transmission models.

1) Constant Rate Transmission Model: The formulation of the optimization problem for constant rate transmission model is as follows:

minimize

$$\sum_{n=1}^{N} t^{(n)} \tag{4}$$

subject to

$$x_l^{(n)} t^{(n)} \geq \alpha_l^{(n)} R_l, \quad l \in [1, L], n \in [1, N] \tag{5}$$

$$\sum_{n=1}^{N} \alpha_l^{(n)} \geq G_l, \quad l \in [1, L] \tag{6}$$

$$p_l^{(n)} \leq p_{\max}, \quad l \in [1, L], n \in [1, N] \tag{7}$$

$$\frac{p_l^{(n)} g_{ll}}{N_0 + \sum_{k \neq l} p_k^{(n)} g_{kl}} \geq \beta_l 1_{\left\{p_l^{(n)} > 0\right\}}, \quad l \in [1, L], n \in [1, N] \tag{8}$$

$$x_l^{(n)} = 1_{\left\{p_l^{(n)} > 0\right\}} r, \quad l \in [1, L], n \in [1, N] \tag{9}$$

$$a_{lk} + 1_{\left\{p_l^{(n)} > 0\right\}} + 1_{\left\{p_k^{(n)} > 0\right\}} \leq 2, \quad l, k \in [1, L], n \in [1, N] \tag{10}$$

variables

$$t^{(n)} \geq 0,\ p_l^{(n)} \geq 0,\ x_l^{(n)} \geq 0,\ \alpha_l^{(n)} \in [0, G_l], \quad l \in [1, L],\ n \in [1, N] \tag{11}$$

where N is the number of time slots, a_{lk} is a constant that takes the value 1 if links l and k share a common node and 0 otherwise. The variables of the optimization problem are $t^{(n)}$, the length of the n-th time slot; $p_l^{(n)}$, the transmit power of link l in time slot n; $x_l^{(n)}$, the transmission rate of link l in time slot n and $\alpha_l^{(n)}$, the number of packets sent by link l in time slot n.

The goal of the optimization problem is to minimize the length of the schedule. Equation (5) represents the packetized transmission requirement where each node transmits an integer number of packets within each time slot. Equation (6) gives the constraint on the total number of packets generated at each node. Note that in the case where the packetized transmission within each time slot is not taken into account, (5) and (6) are replaced by one equation only representing the total number of bits that needs to be transmitted by each node as $\sum_{n=1}^{N} x_l^{(n)} t^{(n)} \geq G_l R_l$. Equation (7) states the upper bound for the transmit power of the links. Equation (8) represents the condition that the SINR of link l is above a fixed threshold if it is active. Equation (9) represents the transmission at fixed rate r for the active links. Equation (10) states that any node in the network cannot transmit to and receive from more than one node simultaneously.

This optimization problem is a Mixed Integer Non-Linear Programming problem thus difficult to solve for the global optimum [31]. We prove the hardness of the problem next.

Theorem 1: The packetized minimum length schedule problem for constant rate transmission model is NP-hard.

Proof: The packetized minimum length schedule problem as formulated by (4)–(11) for constant rate transmission model is equivalent to the integrated link scheduling and power control problem in [12], which is shown to be NP-hard, for the instance in which the packet length of all the links in the network is the same. □

2) Continuous Rate Transmission Model: The formulation of the optimization problem for continuous rate transmission model is very similar to that formulated for constant rate transmission model except that (8) and (9) are replaced by the following constraint:

$$x_l^{(n)} \leq W \times \log\left(1 + \frac{1}{W} \times \frac{p_l^{(n)} g_{ll}}{N_0 + \sum_{k \neq l} p_k^{(n)} g_{kl}}\right), \quad l \in [1, L], n \in [1, N] \tag{12}$$

This problem is a non-convex optimization problem for which there is no known polynomial time algorithm [31].

3) Discrete Rate Transmission Model: The formulation of the optimization problem for discrete rate transmission model is also very similar to that formulated for constant rate transmission model except that (8) and (9) are replaced by the following constraints:

$$p_l^{(n)} g_{ll} - z_{li}^{(n)} \gamma^i \left(N_0 + \sum_{k \neq l} p_k^{(n)} g_{kl}\right) \geq 0, \quad l \in [1, L], n \in [1, N], i \in [1, M] \tag{13}$$

$$\sum_{i=1}^{M} z_{li}^{(n)} = 1, \quad l \in [1, L], n \in [1, N] \tag{14}$$

$$x_l^{(n)} = \sum_{i=1}^{M} z_{li}^{(n)} r^i, \quad l \in [1, L], n \in [1, N] \tag{15}$$

and the inclusion of the variable $z_{li}^{(n)} \in \{0,1\}$, where $l \in [1, L]$, $n \in [1, N]$, $i \in [1, M]$, that takes value 1 if link l transmits at rate r^i in time slot n and 0 otherwise. Equation (13) states that the assigned rate r^i to link l satisfies the corresponding SINR requirement as $\gamma_l^{(n)} \geq \gamma^i$. Equations (14) and (15) state that each link l is assigned to exactly one rate.

Corollary 1: The packetized minimum length schedule problem for discrete rate transmission model is NP-hard.

Proof: Discrete rate transmission model is equivalent to constant rate transmission model for the instance in which $M = 2$. Since the packetized minimum length schedule problem for constant rate transmission model is NP-hard based on Theorem 1, the packetized minimum length schedule problem for discrete rate transmission model is NP-hard. □

B. Solution Strategy

The exponential complexity of the packetized minimum length schedule problem for different transmission models necessitates the use of heuristic algorithms. The solution strategy proposed in this paper is based on the decomposition of the power control and rate adaptation, and scheduling problems as described below:

- The constraints of the joint power control, rate adaptation and scheduling problem provided in equations (5)–(15) include the variables for the transmit power and rate of the links and the slot length corresponding to only one time slot at a time except the constraint on the total number of packets generated at each node given in (6). Therefore, given the number of packets to be transmitted by each link, the length of the time slot should be minimized independent of the other slots in order to attain minimum total schedule length. A novel power control and rate adaptation problem formulated as the minimization of the time slot length required for the concurrent transmission of a set of simultaneously active links each transmitting an integer number of packets is formulated and solved for constant rate, continuous rate and discrete rate transmission models.
- The optimal scheduling contains the time slots each of which corresponds to a subset of the links with each link in the subset sending a certain number of packets. Therefore, given the minimum time slot lengths corresponding to all the subset of the links concurrently transmitting any possible number of packets, calculated by using the formulation of the optimal power control and rate adaptation problem described previously, the scheduling problem is formulated as an Integer Programming (IP) problem where the number of variables is exponential in the number of the links.
- To solve the large-scale IP formulation for the scheduling problem, Branch and Price Method and Column Generation Method based heuristics are proposed.

IV. Power Control and Rate Adaptation Problem

The power control and rate adaptation problem aims to minimize the length of the time slot given the number of packets to be transmitted by each active link in that time slot. Given the power allocation of the links in a time slot, the feasible and achievable rate region for a link given in equations (9), (12) and (15) for constant, continuous and discrete rate transmission models respectively is independent of the rate of the concurrently transmitting links. Since higher rate results in smaller transmission delay, the optimal transmission rate is the maximum achievable rate for the minimization of the time slot length. Power control and rate adaptation problem can therefore be reduced to a pure power control problem where the transmission rates of the links are formulated as a function of the transmit powers of the links.

Let us define the set of active links in a time slot S. We assume that none of the links in the set S have common end point to satisfy the requirement given in (10). Let t be the length of the time slot to be allocated to the transmission of α_l packets by each link l within the set S. The objective of the optimal power control problem under all transmission models is to minimize t. We remove the superscript of the variables, which represent the time slot in Section III, to simplify the expressions in this section.

A. Constant Rate Transmission Model

The optimal power control problem for constant rate transmission model is formulated as

minimize

$$t \tag{16}$$

subject to

$$\frac{\alpha_l R_l}{x_l} \leq t, \quad l \in S \tag{17}$$

$$p_l \leq p_{\max}, \quad l \in S \tag{18}$$

$$\frac{p_l g_{ll}}{N_0 + \sum_{k \neq l} p_k g_{kl}} \geq \beta_l, \quad l \in S \tag{19}$$

$$x_l = r, \quad l \in S \tag{20}$$

variables

$$t \geq 0, \; p_l \geq 0, \quad l \in S \tag{21}$$

The constraints given in equations (17)–(20) correspond to the constraints given in (5), (7), (8), and (9), respectively.

The power control problem for constant rate transmission model first requires testing the feasibility of the concurrent transmission of the links in S. For the feasibility of the concurrent transmission of the links in S, there should exist a set of transmit power allocations to the links in the set S such that the SINR constraint of each link $l \in S$ as given in (19) is satisfied while meeting the maximum transmit power constraint in (18). The existence of such a set of transmit power allocations is determined by testing Perron-Frobenius conditions [32] described as follows: Let B_S be $|S| \times |S|$ relative channel gain matrix such that the element in the l-th row and k-th column of B_S takes value g_{kl}/g_{ll} for $l \neq k$ and 0 for $l = k$. Let D_S be $|S| \times |S|$ diagonal matrix with the l-th diagonal entry equal to β_l. Let V_S be $|S| \times 1$ normalized noise power vector with the

l-th element equal to $\beta_l N_0/g_{ll}$. Perron-Frobenius conditions state that there exists a feasible power vector if and only if the largest real eigenvalue of $D_S B_S$ is less than 1 and every element of the component-wise minimum power vector $(I - D_S B_S)^{-1} V_S$ is less than or equal to $p_{\max}$. If the concurrent transmission of the links within the set S is feasible, then the optimal value of t is equal to $\max_{l\in S}(\alpha_l R_l/r)$.

Since the complexity of evaluating Perron-Frobenius conditions for $|S|$ links is $O(|S|^3)$[32], the complexity of solving the power control problem for constant rate transmission model is $O(|S|^3)$.

B. Continuous Rate Transmission Model

The optimal power control problem for continuous rate transmission model is very similar to that formulated for constant rate transmission model except equations (19) and (20) are replaced by the constraint given in equation (12) for the links in the set S. This power control problem is a non-convex optimization problem for which there is no known polynomial time algorithm [31]. In the following, we first state the theorem on the equality of the optimal time duration required for the transmission of all the links assigned to the same time slot then provide an iterative algorithm that gets close to the optimal time slot length within a certain accuracy exponentially fast.

Theorem 2: Let t^* denote the optimal length of the time slot allocated to the transmission of α_l packets by each link l within the set S such that $t^* = \max_{l\in S} t_l^*$ where t_l^* is the transmission time required for link l. Then, the transmission times of all the links within S are equal; i.e., $t^* = t_l^* \; \forall\, l \in S$.

Proof: We will prove the theorem by contradiction. Suppose that $t^* \neq t_l^*$ for an arbitrary $l \in S$. Let $t_k^* = \max_{j\in S} t_j^*$. Then $t_k^* > t_l^*$. If we decrease the transmission power of link l by an arbitrarily small amount such that t_l^* is still less than t_k^*, the transmission time of all the links except link l decreases due to the decreasing amount of interference created by node l. Note that the transmission rate of a link j is an increasing function of the transmission power of link j and decreasing function of the transmission power of link $i \neq j$. As a result, time slot length required for the transmission of all the links, $t^* = \max_{j\in S} t_j^*$ decreases. Hence, as long as there exists a node j such that $t_j < \max_{l\in S} t_l$, we can improve the solution by decreasing the transmission power of link j. Hence, at the optimal solution, the transmission time of every link in the set S must be equal. □

If we fix the value of the length of the allocated time slot, the optimal power control problem reduces to determining whether there exists a set of transmit power assignment to the links in the set S such that the SINR constraint of each link $l \in S$ as given in

$$\frac{p_l g_{ll}}{N_0 + \sum_{k\neq l, k\in S} p_k g_{kl}} \geq \beta_l \tag{22}$$

where

$$\beta_l = W\left(2^{\alpha_l R_l/(Wt)} - 1\right) \tag{23}$$

is satisfied while meeting the maximum transmit power constraint in (18). The existence of such a set of transmit power vector is determined by testing Perron-Frobenius conditions as explained in Section IV-A. Fast-Iterative Power Control Algorithm (FIPCA) is based on performing an intelligent search for the minimum feasible value of t.

FIPCA algorithm determines an interval for the optimal value of the length of the time slot corresponding to the transmission of α_l packets by each link l within the feasible set S and then reduces the length of this interval by half in each iteration. The lower and upper end points of this interval determined as the lowest infeasible value and highest feasible value for the time slot length t are denoted by t^{lw} and t^{up} respectively. The algorithm starts by setting the lower bound to the maximum value of the time slot lengths required when only one link transmits at maximum transmit power $p_{\max}$ and upper bound to the length of the time slot when every link $l \in S$ transmits at maximum transmit power $p_{\max}$ (Lines 1–2). In each iteration, the algorithm checks the feasibility of the middle point of the interval (t^{lw}, t^{up}) (Lines 4–5). If the value $t = (t^{lw} + t^{up})/2$ is feasible then any value greater than t is a worse feasible solution than t thus requiring the search for the optimal time slot over interval $(t^{lw}, (t^{lw} + t^{up})/2)$ by updating the upper bound (Lines 6–7). On the other hand, if the value $t = (t^{lw} + t^{up})/2$ is infeasible then any value less than t is also infeasible requiring the search for the optimal time slot over interval $((t^{lw} + t^{up})/2, t^{up})$ by updating the lower bound (Lines 8–9). The termination criteria of the algorithm is achieving a predetermined relative error bound level represented by ϵ (Line 3). The value of the time slot length is then set to t^{up} (Line 12). Since the optimal time slot length t^{opt} is between t^{lw} and t^{up}, the relative error bound of the optimal time slot length given by $(t^{up} - t^{opt})/t^{opt}$ is less than $(t^{up} - t^{lw})/t^{lw}$ so less than ϵ meaning that the value picked by the algorithm t^{up} lies in the interval $[t^{opt}, (1+\epsilon)t^{opt}]$.

Algorithm 1 Fast Iterative Power Control Algorithm (FIPCA)

1: $t^{lw} = \max_{l\in S}(\alpha_l R_l/(W \times \log(1+(1/W)\times(p_{\max} g_{ll}/N_0))))$;
2: $t^{up} = \max_{l\in S}(\alpha_l R_l/(W \times \log(1+(1/W)\times(p_{\max} g_{ll}/(N_0 + \sum_{k\neq l,k\in S} p_{\max} g_{kl})))))$
3: **while** $((t^{up} - t^{lw})/t^{lw}) > \epsilon$ **do**
4: $\quad t = (t^{up} + t^{lw})/2$;
5: $\quad$ check feasibility for time-slot length t;
6: $\quad$ **if** t is feasible **then**
7: $\quad\quad t^{up} = t$;
8: $\quad$ **else**
9: $\quad\quad t^{lw} = t$;
10: $\quad$ **end if**
11: **end while**
12: $t = t^{up}$;

Lemma 1: For a fixed predetermined relative error bound level ϵ, the complexity of the FIPCA algorithm is $O(K|S|^3)$, where $K = \lceil \log((t_{init}^{up} - t_{init}^{lw})/\epsilon t_{init}^{lw}) \rceil$, t_{init}^{up} and t_{init}^{lw} are the initial values of the upper and lower bounds for the optimal time slot length set by the algorithm (Lines 1–2).

Proof: The FIPCA algorithm reduces the initial ratio $(t^{up}_{init} - t^{lw}_{init})/t^{lw}_{init}$ by a factor of at least two in each iteration since $t^{up} - t^{lw}$ is reduced by half by setting either t^{up} or t^{lw} to the middle point of the interval $[t^{up}, t^{lw}]$ (Lines 4–9). Hence, the number of iterations required to decrease the ratio $(t^{up} - t^{lw})/t^{lw}$ to ϵ is $K = \lceil \log((t^{up}_{init} - t^{lw}_{init})/\epsilon t^{lw}_{init}) \rceil$. Since the algorithm checks the feasibility of the existence of a transmit power vector satisfying SINR and maximum power constraints in each iteration evaluating the Perron-Frobenius conditions (Line 5) of $O(|S|^3)$ complexity, the complexity of the algorithm is $O(K|S|^3)$. □

C. Discrete Rate Transmission Model

The optimal power control problem for discrete rate transmission model is very similar to that formulated for constant rate transmission model except Equations (19) and (20) are replaced by the constraints given in Equations (13)–(15) for the links in the set S. This power control problem is a Mixed Integer Non-linear Programming thus difficult to solve for the global optimum [31]. Note that unlike the continuous rate transmission model, the optimal time duration required for the transmission of the links assigned to the same time slot are not necessarily equal in this case due to the discrete nature of the rates.

A straightforward search algorithm enumerates all possible rate allocations of the links in the set S and check their feasibility by testing the existence of a power vector satisfying the SINR requirements corresponding to these rate allocations and maximum power constraints by evaluating Perron-Frobenius conditions. The optimal solution is then the minimum of the time slot lengths of the feasible rate allocations. The complexity of this search is $\mathcal{O}(M^{|S|}|S|^3)$.

We will now propose a fast fathoming-based smart enumeration algorithm based on the following two lemmas. In this algorithm, we use a tree structure for the rate index vector $x_r = (i_1, i_2, \ldots, i_{|S|})$ where the links are enumerated from 1 to $|S|$ and the j-th link is assigned to the i_j-th rate level where $i_j \in [1, M]$. The root of the tree is $(M, M, \ldots, M)$. The children of the rate index vector $x_r = (i_1, i_2, \ldots, i_{|S|})$ are obtained by updating the j-th element of x_r as $i_j - 1$ and keeping the remaining elements the same for every $j \in [1, |S|]$ such that $i_j > 2$. A rate index vector $(i_1, i_2, \ldots, i_{|S|})$ is a descendant of another rate index vector $(m_1, m_2, \ldots, m_{|S|})$ if $i_j \le m_j$ for every $j \in [1, |S|]$.

Lemma 2: If a rate index vector $x_r = (i_1, i_2, \ldots, i_{|S|})$ is feasible with the corresponding time slot length t_r then the time slot length corresponding to any descendant of x_r is greater than or equal to t_r.

Proof: Let us denote the descendant of x_r by $y_r = (p_1, p_2, \ldots, p_{|S|})$. Since $p_j \le i_j$ for $j \in [1, |S|]$, the transmission duration of link j at rate r^{p_j} is greater than or equal to that at rate r^{i_j} for every $j \in [1, |S|]$. Since the time slot length is the maximum of the transmission duration of all the links in the set S, the time slot length corresponding to any descendant of x_r is greater than or equal to t_r. □

Lemma 3: If a rate index vector $x_r = (i_1, i_2, \ldots, i_{|S|})$ is infeasible and $\max_{j\in[1,|S|]}(\alpha_j R_j / r^{i_j})$ is greater than or equal to the time slot length corresponding to a feasible rate index vector denoted by t_f, then the time slot length corresponding to any feasible descendant of x_r is greater than or equal to t_f.

Proof: Let us denote the descendant of x_r by $y_r = (p_1, p_2, \ldots, p_{|S|})$. Since $p_j \le i_j$ for $j \in [1, |S|]$, the time slot length corresponding to y_r that is given by $\max_{j\in[1,|S|]}(\alpha_j R_j / r^{p_j})$ is greater than or equal to $\max_{j\in[1,|S|]}(\alpha_j R_j / r^{i_j})$, which is greater than or equal to t_f. □

Algorithm 2 Fast Tree-Based Fathoming Algorithm (FTFA)

1: $A = \{(M, M, \ldots, M)\}$;
2: $F = \emptyset$;
3: $C = \emptyset$;
4: $t^{opt} = \infty$;
5: $x_r^{opt} = \emptyset$;
6: **while** $A \ne \emptyset$ **do**
7: $x_r = (i_1, i_2, \ldots, i_{|S|})$ = first rate index vector in A;
8: discard x_r from set A;
9: **if** x_r is not descendant of a node in F and not inside C **then**
10: $t = \max_{j\in[1,|S|]}(\alpha_j R_j / r^{i_j})$;
11: **if** x_r is feasible **then**
12: **if** $t < t^{opt}$ **then**
13: $t^{opt} = t$;
14: $x_r^{opt} = x_r$;
15: **end if**
16: add x_r to set F; {due Lemma 2}
17: **else**
18: **if** $x_r = (2, 2, \ldots, 2)$ **then**
19: break; {no feasible solution exists}
20: **end if**
21: **if** $t >= t^{opt}$ **then**
22: add x_r to set F; {due Lemma 3}
23: **else**
24: include children of x_r at the beginning of set A;
25: include x_r in set C;
26: **end if**
27: **end if**
28: **end if**
29: **end while**

Based on the foregoing lemmas, we propose the Fast Tree-Based Fathoming Algorithm (FTFA) as described next. x_r^{opt} and t^{opt} correspond to the optimal rate index vector and optimal time slot length in each iteration respectively. x_r^{opt} and t^{opt} are initialized to $\emptyset$ and ∞ respectively (Lines 4–5). Three sets are updated in each iteration of the algorithm. A is the set of rate index vectors to be evaluated in order. A is initialized to $\{(M, M, \ldots, M)\}$ (Line 1) and extended to include the children of the infeasible rate index vector for which $t = \max_{j\in[1,|S|]}(\alpha_j R_j / r^{i_j}) < t^{opt}$ since these children are potential candidates for optimal rate index vectors (Line 24). F is initialized to $\emptyset$ (Line 2) and extended to include feasible rate index vectors since any descendant of these vectors

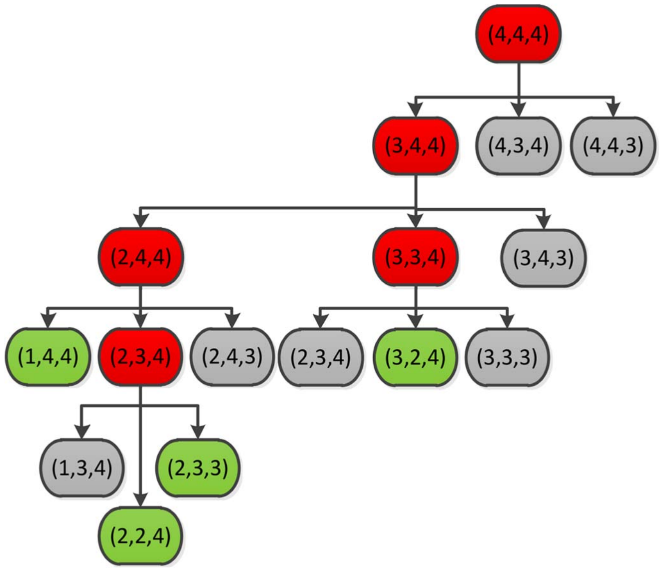

Fig. 2. FTFA algorithm illustration on the tree structure for the case where $|S| = 3$ and $M = 4$. The rate vectors are evaluated in the following order: (4,4,4), (3,4,4), (2,4,4), (1,4,4), (2,3,4), (1,3,4), (2,2,4), (2,3,3), (2,4,3), (3,3,4), (2,3,4), (3,2,4), (3,3,3), (3,4,3), (4,3,4), and (4,4,3). Green-colored rate index vectors are the fathomed vectors that are evaluated as feasible and the descendants of which are pruned by the algorithm based on Lemma 2 (Line 16). Grey-colored rate index vectors are the vectors that are not evaluated since they are either a descendant of a node in F or inside C (Line 9) or fathomed since they are infeasible rate index vectors for which t is greater than or equal to the current optimal time slot length t^{opt} based on Lemma 3 (Lines 21–22). Red-colored rate index vectors are the vectors that are evaluated as infeasible for which t is less than the current optimal time slot length t^{opt} and whose child nodes are generated as a consequence (Lines 24–25). For example, vector (2,4,3) is fathomed since the corresponding t value is higher than the current optimal time slot length associated with the best feasible rate vector (2,2,4), vector (1,3,4) is not evaluated since it is a descendant of a previously fathomed vector (1,4,4) and vector (2,3,4) which is generated as a child node of rate vector (3,3,4) is not evaluated since it is inside C.

is also feasible and cannot give a time slot length smaller than the current time slot length t due to Lemma 2 (Line 16) and infeasible rate index vectors for which t is greater than or equal to the current optimal time slot length t^{opt} since any descendant of these vectors cannot give a time slot length smaller than t^{opt} due to Lemma 3 (Lines 21–22). C is the set of rate index vectors that are evaluated but not fathomed. C is initialized to $\emptyset$ (Line 3). The children of the rate index vector placed into C in each iteration are included in A to be checked in the following iterations (Line 25). In each iteration of the algorithm, the first rate index vector from the set A is chosen and evaluated for feasibility by checking Perron-Frobenius conditions if it is not descendant of a node in F and not inside C (Lines 7–9). If the rate index vector evaluated in the current iteration is feasible, its corresponding time slot length is checked for optimality (Lines 11–12). If the time slot length is less than the current optimal value t^{opt}, x_r^{opt}, and t^{opt} are updated (Lines 13–14). If there are no rate index vectors to be evaluated in A (Line 6) or the smallest rate index vector where all the nodes in the set S are active, i.e., $x_r = (2, 2, \ldots, 2)$, is infeasible (Lines 18–20), the algorithm stops. Fig. 2 illustrates the FTFA algorithm through an example.

The complexity of FTFA algorithm is $O(M^{|S|}|S|^3)$ since the algorithm checks the feasibility of all of the $M^{|S|}$ possible rate index vectors by evaluating Perron-Frobenius conditions with complexity $O(|S|^3)$ in the worst case. However, the average runtime of FTFA is much smaller due to the fathoming mechanism as will be demonstrated via extensive simulations in Section VIII.

V. Optimal Scheduling Problem Formulation

Once the minimum time slot length of all possible simultaneous packet transmission scenarios, i.e., the transmission of α_l packets by link l where $\alpha_l \in [0, G_l]$ for $l \in [1, L]$, is determined by solving the power control and rate adaptation problem described in Section IV, the minimum length scheduling problem can be formulated by assigning a variable to each of these scenarios as described next.

Let $\mathcal{E} = \{\mathcal{E}_k : 1 \leq k \leq |\mathcal{E}|\}$ denote the set of all feasible simultaneous packet transmission scenarios of the link set $\mathcal{L} = \{1, 2, \ldots, L\}$. Note that $|\mathcal{E}| \leq \prod_{i=1}^{L}(G_i + 1)$ with equality for the case where no two links share any common node.

Let Q denote an $L \times |\mathcal{E}|$ matrix such that the element in the l-th row and k-th column of Q denoted by q_{lk} is the number of packets transmitted by link l in the k-th simultaneous packet transmission scenario in $\mathcal{E}$, i.e., $\mathcal{E}_k$.

The minimum length scheduling problem is then formulated as an Integer Programming (IP) problem [33] as

$$\textbf{\textit{minimize}} \quad t^T c \tag{24}$$

$$\textbf{\textit{subject to}} \quad Qc \geqslant G \tag{25}$$

$$\textbf{\textit{variables}} \quad c^{(k)} \in \{0, 1\}, \ k \in [1, |\mathcal{E}|] \tag{26}$$

where t is $|\mathcal{E}| \times 1$ vector whose k-th element $t^{(k)}$ is the minimum time slot length corresponding to the k-th simultaneous packet transmission scenario $\mathcal{E}_k$, G is $L \times 1$ vector whose k-th element is the packet requirement of the k-th link, i.e., G_k. The variable of the IP problem is the $|\mathcal{E}| \times 1$ vector c whose k-th element denoted by $c^{(k)}$ takes value 1 if the transmission scenario $\mathcal{E}_k$ exists in the optimal schedule and 0 otherwise. Equation (25) represents the packet requirements of the links in the network.

There arise two fundamental difficulties in solving this IP problem. First, it requires an exponential effort to determine the time slot length required for each possible transmission scenario. Second, even if the time slot lengths are determined, there are exponential number of integer variables in the foregoing IP formulation which makes the problem intractable. Since there are $\prod_{i=1}^{L}(G_i + 1)$ possible transmission scenarios, the complexity of determining the time slot length corresponding to all transmission scenarios is $O(\prod_{i=1}^{L}(G_i + 1) f_p)$ where f_p is the complexity of the power control algorithm used depending on the transmission rate model and equal to $O(|L|^3)$, $O(|L|^3)$ and $O(M^{|L|}|L|^3)$ for constant, continuous and discrete rate transmission models as derived in Sections IV-A–IV-C respectively. The complexity of solving the IP problem with $\prod_{i=1}^{L}(G_i + 1)$ variables given the corresponding time slot lengths on the other hand is $O(2^{\prod_{i=1}^{L}(G_i+1)})$. The complexity of solving the optimal scheduling problem is therefore $O(\prod_{i=1}^{L}(G_i+1)f_p + 2^{\prod_{i=1}^{L}(G_i+1)})$.

VI. Branch and Price Method Based Heuristic Algorithm for Scheduling

Branch and Price Method Based Heuristic Algorithm (BPH) developed to overcome the intractability problem of the IP formulation in Section V is based on using Column Generation Method based heuristic algorithm to solve the LP relaxation of the IP problem at each node of the Branch-and-Bound tree.

BPH starts with the exponential IP formulation given in (24)–(26). The LP relaxation of the exponential IP formulation is solved using the Column Generation Method (CGM). If the solution is fractional, one of the fractional variables is picked and two new IP problems are formed by setting the value of the corresponding variable to 0 and 1, which we call branching forming new nodes in the tree. After branching, each resulting node has an associated IP formulation. Note that at each node, the associated IP formulation does not contain some of transmission scenarios in $\mathcal{E}$ since the associated variables to these transmission scenarios are either set to 0 or 1 through the branch on which the particular node is located. We iteratively solve the LP relaxation of the IP formulations at the nodes in the tree using CGM. If the CGM does not provide a feasible solution at a certain node, we stop branching (fathom) that node. If the solution of the CGM is integral, we update the best integral solution and fathom that node. If the solution is fractional and the solution is worse than the best integral solution, we also fathom that node; otherwise we continue branching by using one of the fractional variables and setting its value to 0 and 1 resulting in two new IP problems, i.e., nodes. This process continues until every node is fathomed.

Column Generation Method (CGM), which is an elegant method generally used for solving large-scale LP problems [33], is used to solve the LP-relaxation of the IP problem associated with the nodes of the Branch and Price tree. CGM decomposes the resulting LP problem into two sub-problems: Restricted Master Problem and Pricing Problem.

A. Restricted Master Problem

The Restricted Master Problem (RMP) is similar to the original problem except that only a small subset of the transmission scenarios $\mathcal{E}^{(s)} \subset \mathcal{E}$ is considered as

$$\textit{minimize} \quad \left(t^{(s)}\right)^T c \tag{27}$$

$$\textit{subject to} \quad Q^{(s)}c \geqslant G \tag{28}$$

$$\textit{variables} \quad 0 \leq c^{(k)} \leq 1,\ k \in [1, |\mathcal{E}^s|] \tag{29}$$

where $t^{(s)}$ and $Q^{(s)}$ only contain the timeslot lengths in vector t and the columns of the transmission scenario matrix Q corresponding to the subsets in $\mathcal{E}^s$ respectively.

To start CGM, we need to choose an initial set $\mathcal{E}^{(s)}$ that guarantees the feasibility of the RMP. A proper choice of $\mathcal{E}^{(s)}$ contains the transmission scenarios where only one link becomes active at a time transmitting all its packets resulting in $t^{(s)}$ as a vector containing the corresponding minimum time slot lengths and $Q^{(s)}$ as an $L \times L$ diagonal matrix whose diagonal elements are the packet requirements of the links.

We can solve RMP and its dual problem with simplex method in polynomial time to obtain its primal optimal solution c^p and dual optimal solution c^d. Since we consider only an arbitrary subset $\mathcal{E}^{(s)} \subset \mathcal{E}$, c^p is not necessarily the optimal solution to the original problem given in (24)–(26). In the original problem, the cost coefficient of each variable $c^{(k)}$ in the objective function is $t^{(k)}$ for $k \in [1, |\mathcal{E}|]$. Thus the reduced cost of a column $q^{(k)}$ in the matrix Q but not in $Q^{(s)}$ is

$$r^{(k)} = t^{(k)} - (c^d)^T q^{(k)} \tag{30}$$

where $t^{(k)}$ is the minimum length of the time slot for the transmission scenario $\mathcal{E}_k$ corresponding to the column $q^{(k)}$.

If there exists a column $q^{(k)}$ the reduced cost of which denoted by $r^{(k)}$ is less than 0, the objective function of the RMP can be further reduced by including the column $q^{(k)}$ in the matrix $Q^{(s)}$. Otherwise, the current solution c^p is the optimal solution to the original problem. Instead of calculating the reduced cost of each column $q^{(k)}$ in the matrix Q but not in $Q^{(s)}$, which requires an exponential effort, we can solve the following Pricing Problem which will generate the column with the minimum reduced cost among all columns in Q matrix.

B. Pricing Problem

The goal of formulating the Pricing Problem (PP) is to find a transmission scenario that can improve the objective of the RMP when included in $Q^{(s)}$ matrix as a column. The objective of the PP formulation is given as

$$t - \sum_{l=1}^{L} c_l^d \alpha_l \tag{31}$$

where c_l^d is the l-th element of the dual optimal solution of the RMP c^d; α_l is the number of packets transmitted by link l; t, the length of the time slot corresponding to the transmission scenario represented by the number of packets α_l transmitted by each link l where $l \in [1, L]$. The constraints and variables of the optimization problem are the same as those of the power control problem provided in Section IV for all transmission models with the additional variable α_l, $l \in [1, L]$.

The PP formulation is a Mixed-Integer Programming problem for which there is no known polynomial-time algorithm [33]. We therefore propose the following heuristic algorithm to efficiently and rapidly solve PP.

C. Heuristic Algorithm for Pricing Problem

The heuristic algorithm called Reduced Cost Minimization Algorithm (RCMA) is based on iteratively including links and corresponding number of packet transmissions one at a time such that the reduced cost of the resulting set is minimized.

Let S and α^S denote the set of links and the number of packet transmissions by the links in the set S respectively such that each link $l \in S$ transmits α_l packets, i.e. $\alpha^S = \{\alpha_{S(1)}, \alpha_{S(2)}, \ldots, \alpha_{S(|S|)}\}$ where $S(i)$ is the i-th element of the set S. The reduced cost of the transmission scenario represented by the pair (S, α^S) is then defined as

$$r^{(S,\alpha^S)} = t^{(S,\alpha^S)} - \sum_{l \in S} c_l^d \alpha_l \tag{32}$$

where $t^{(S,\alpha^S)}$ is the length of the time slot corresponding to the transmission scenario where the nodes in the set S transmit the number of packets in the set α^S.

RCMA algorithm iteratively includes the links and corresponding number of packet transmissions in the initially empty sets S and α^S respectively (Line 1) such that the link and corresponding number of packet transmissions minimize the reduced cost of the resulting transmission scenario after their addition (Lines 3–5). The algorithm stops when the set S includes all the links in $\mathcal{L}$ (Line 2) or the addition of a link does not decrease the reduced cost of the transmission scenario represented by (S, α^S) (Lines 6–8).

Algorithm 3 Reduced Cost Minimization Algorithm (RCMA)

1: $S = \emptyset$, $\alpha^S = \emptyset$, $D = \mathcal{L}$;
2: **while** $S \neq \mathcal{L}$ **do**
3: **if** $\min_{l \in D, \alpha_l \in [1, G_l]} r^{(S+\{l\}, \alpha^S+\{\alpha_l\})} < r^{(S,\alpha^S)}$ **then**
4: $(k, \alpha_k) = \arg\min_{l \in D, \alpha_l \in [1, G_l]} r^{(S+\{l\}, \alpha^S+\{\alpha_l\})}$;
5: $S = S + \{k\}$, $D = D - \{k\}$, $\alpha^S = \alpha^S + \{\alpha_k\}$;
6: **else**
7: **break**;
8: **end if**
9: **end while**

D. Complexity of BPH

BPH investigates all the nodes in the branch-and-price tree in the worst case. The total number of the nodes in the branch-and-price tree is less than $2^{\prod_{i=1}^{L}(G_i+1)+1}$ since each node is obtained by setting a subset of $\prod_{i=1}^{L}(G_i + 1)$ variables to either 0 or 1. On the other hand, at each node of the branch-and-price tree, CGM is used to solve the LP relaxation of the associated IP formulation. The maximum number of iterations in the CGM is equal to the maximum number of columns CGM can generate, which is equal to the total number of transmission scenarios, i.e., $\prod_{i=1}^{L}(G_i + 1)$. In each iteration of CGM, the RMP solves an LP problem of at most $\prod_{i=1}^{L}(G_i + 1)$ variables resulting in the worst case complexity of $O(L(\prod_{i=1}^{L}(G_i + 1))^4 d)$ using Interior Method [34] where d is the maximum number of bits used to represent the coefficients of the objective function and constraints of the optimization problem, i.e., t^k, q_{lk} and G_l, for $l \in [1, L]$ and $k \in [1, \prod_{i=1}^{L}(G_i + 1)]$. The RCMA algorithm solving the PP of the CGM in each iteration has complexity of $O(\sum_{i=1}^{L}(G_i))$ provided that the time slot length corresponding to all transmission scenarios is generated once in the worst case with complexity $O(\prod_{i=1}^{L}(G_i + 1)f_p)$. Therefore, the overall complexity of BPH is $O(\prod_{i=1}^{L}(G_i + 1)f_p + 2^{\prod_{i=1}^{L}(G_i+1)+1}(L(\prod_{i=1}^{L}(G_i + 1))^5 d))$. Note that the worst case complexity of BPH is much higher than that of the optimal IP formulation. However, since the fathoming mechanism prevents the generation of all the nodes in the branch-and-price tree and CGM only generates a subset of the columns, the average runtime of BPH is less than that of the optimal IP formulation as will be demonstrated via extensive simulations in Section VIII.

VII. Column Generation Method Based Heuristic Algorithms for Scheduling

BPH described in Section VI explores the whole branch-and-bound tree by solving the LP relaxation of the IP problem at each node using the Column Generation Method (CGM) based heuristic algorithm. We now propose heuristic methods of lower complexity that still exploit the CGM solution given in Sections VI-A and VI-B.

A. Restricted Master Heuristic

Restricted Master Heuristic (RMH) first solves the LP relaxation of the optimal IP formulation generating the columns of the Q matrix required for the optimal fractional solution then formulates the IP problem with these columns. The algorithm starts with the exponential IP formulation given in (24)–(26). The LP relaxation of the exponential IP formulation is solved using the Column Generation Method (CGM). In CGM, the set of transmission scenarios $\mathcal{E}^{(s)}$ is initialized to guarantee the feasibility of the RMP and extended by including the transmission scenario that minimizes the reduced cost by the use of the PP at each iteration of CGM. The resulting set $\mathcal{E}^{(s)}$, the corresponding time slot length vector $t^{(s)}$ and transmission scenario matrix $Q^{(s)}$ are then input to the original IP formulation given in (24)–(26) by setting $Q = Q^{(s)}$ and $t = t^{(s)}$.

RMH requires solving CGM in $\prod_{i=1}^{L}(G_i + 1)$ iterations with at most $\prod_{i=1}^{L}(G_i + 1)$ variables and the optimal IP formulation with $\prod_{i=1}^{L}(G_i + 1)$ variables in the worst case. The overall complexity of RMH is therefore $O(\prod_{i=1}^{L}(G_i + 1)f_p + L(\prod_{i=1}^{L}(G_i + 1))^5 d + 2^{\prod_{i=1}^{L}(G_i+1)})$. Similar to BPH, the worst case complexity of RMH is much higher than that of the optimal IP formulation. However, since in most cases CGM terminates in much smaller number of iterations than the total number of transmission scenarios given by $\prod_{i=1}^{L}(G_i + 1)$, the average runtime of the RMH is much smaller than that of the optimal IP formulation as will be demonstrated via extensive simulations in Section VIII.

B. Rounding Heuristic

Rounding Heuristic (RH) explores only one branch of the branch-and-price tree. The algorithm starts by solving the

TABLE I
COMPLEXITY COMPARISON OF THE ALGORITHMS

OPT	$O(\prod_{i=1}^{L}(G_i+1)f_p + 2^{\prod_{i=1}^{L}(G_i+1)})$
BPH	$O(\prod_{i=1}^{L}(G_i+1)f_p + 2^{\prod_{i=1}^{L}(G_i+1)+1}(L(\prod_{i=1}^{L}(G_i+1))^5 d))$
RMH	$O(\prod_{i=1}^{L}(G_i+1)f_p + L(\prod_{i=1}^{L}(G_i+1))^5 d + 2^{\prod_{i=1}^{L}(G_i+1)})$
RH	$O(\prod_{i=1}^{L}(G_i+1)f_p + \sum_{i=1}^{L}(G_i)(L(\prod_{i=1}^{L}(G_i+1))^5 d))$

LP-relaxation of the exponential IP formulation given in (24)–(26) using the CGM method. Among the fractional values in the solution, the largest one is chosen setting its value to 1, in contrast to exploring both values of 0 and 1 as in the Branch-and-Price Method. The packet requirements of the links are then updated by considering that the transmission scenario corresponding to the variable is set to 1. In each iteration, the largest fractional value in the solution of the CGM for the LP relaxation of the resulting problem is set to 1 updating the packet requirements of the links accordingly. The algorithm stops when the packet requirements of all the links are met.

RH requires solving CGM on the nodes in one branch of the branch-and-price tree. As the algorithm proceeds on each node of the investigated branch, one of the transmission scenarios is included by setting the corresponding variable to 1, resulting in the transmission of at least one packet. The maximum number of the nodes on the branch is therefore $\sum_{i=1}^{L}(G_i)$. Therefore, the worst case complexity of RH is $O(\prod_{i=1}^{L}(G_i+1)f_p + \sum_{i=1}^{L}(G_i)(L(\prod_{i=1}^{L}(G_i+1))^5 d))$, which is much smaller than that of BPH.

The worst case complexities of the proposed algorithms and the optimal IP formulation (OPT) are summarized in Table I.

VIII. Simulations and Performance Evaluation

The goal of this section is to evaluate the performance of the proposed scheduling algorithms including BPH, RMH and RH together with the power control and rate adaptation algorithms for constant rate, continuous rate and discrete rate transmission models, and compare their performance to that of the Traditional Scheduling (TS) algorithm for constant and discrete rate transmission models. TS algorithm is described as follows: The CGM based scheduling algorithm proposed in [22] is implemented to obtain the optimal schedules neglecting the packet based transmissions. Since the resulting optimal schedules divide the packet transmissions into several data chunks each of which is transmitted in different time slots, the schedules are adapted for packet based transmissions by updating the time slot lengths such that each link transmits an integer number of packets, which is the smallest number greater than or equal to the ratio of the number of bits assigned to that link to the packet length of the link, if the link did not satisfy its packet traffic requirement until that time slot. The performance of the TS algorithm is not included for continuous rate transmission model since no algorithm has been proposed to minimize the schedule length by using the continuous rate transmission model in the literature and CGM cannot be applied to the continuous rate transmission model since the number of columns corresponding to the feasible transmission rates of the links is infinite.

We used MATLAB on a computer with a 2.5 GHz CPU and 4 GB RAM to run the simulations. Simulation results are obtained based on 1000 independent random network topologies where various numbers of links with 1 m fixed length are uniformly distributed over a square area of 3 × 3 meters. The attenuation of the links are determined using the path loss model given by $PL(d) = PL(d_0) - 10n\log_{10}(d/d_0) + Z$, where d is the distance between the transmitter and receiver, d_0 is the reference distance, $PL(d)$ is the path loss at distance d, n is the path loss exponent and Z is a Gaussian random variable with zero mean and σ_z^2 variance. The parameters used in the simulations are $n = 4$, $\sigma_z^2 = 2$ dB^2, $PL(d_0) = 30$ dB, $d_0 = 1$ m, $N_0 = 10^{-8}$ W/Hz, $p_{\max} = 10$ mW, $W = 100$ MHz, $R_l = 100$ bits, G_l is randomly determined from the set {1, 2,3} for each $l \in [1, L]$, $\beta_l = 10$ dB for all $l \in [1, L]$ in constant rate transmission model, $\epsilon = 0.001$ in continuous rate transmission model and $\gamma = (-\infty, 10, 20, 30)$ dB in discrete rate transmission model. The transmission rate corresponding to these SINR values are calculated by using (2).

Fig. 3(a)–(c) show 95% confidence interval bars depicted around the mean of the approximation ratio of the proposed scheduling algorithms including BPH, RMH and RH algorithms and the TS algorithm for constant rate, continuous rate and discrete rate transmission models respectively. The approximation ratio is defined as the ratio of the schedule length obtained by the heuristic algorithm to that obtained by OPT. Note that the number of the links is limited to 20 for constant and continuous rate models, and to 10 for discrete rate models since solving the OPT is intractable for networks with higher number of links. For constant and discrete rate transmission models, the proposed algorithms outperform TS algorithm. We observe similar behavior for all the proposed algorithms under all transmission models: The approximation ratio of BPH is very close to 1, i.e., optimal solution, and robust to the increase in the number of the links whereas the approximation ratio of RMH and RH is worse than that of BPH and increases as the number of the links increases. Moreover, the performance of RH is slightly better than RMH since RH continues to search for the best set of transmission scenarios in each iteration whereas RMH uses only the set of transmission scenarios determined at the output of the first CGM.

Fig. 4(a)–(c) illustrate the average runtime performance of BPH, RMH, RH and OPT scheduling algorithms for constant rate, continuous rate and discrete rate transmission models respectively. For constant and continuous rate transmission models, the average runtime of the RMH and RH scheduling algorithms increases almost linearly in the number of the links due to the greedy structure of these algorithms and linear increase in the runtime of the corresponding power control algorithms. On the other hand, the runtime of the BPH and OPT algorithms is exponential in the number of the links. However, the exponent of the runtime of the OPT and BPH algorithms are around 2 and 1.3 respectively, which creates a significant runtime difference as the number of the links grows larger. Although the runtime of the OPT algorithm is lower than that of the BPH algorithm for up to 20 links, when projected, it is expected that the BPH algorithm would outperform OPT algorithm for 22 links and the runtime of the BPH would be

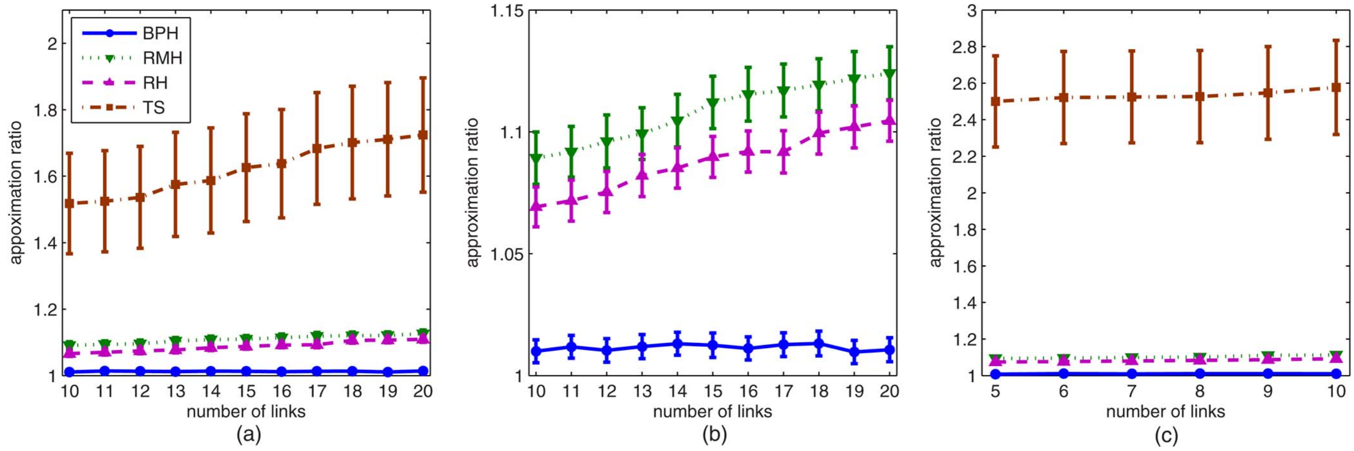

Fig. 3. Approximation ratio of BPH, RMH, RH, and TS scheduling algorithms for a) constant rate, b) continuous rate, c) discrete rate transmission models.

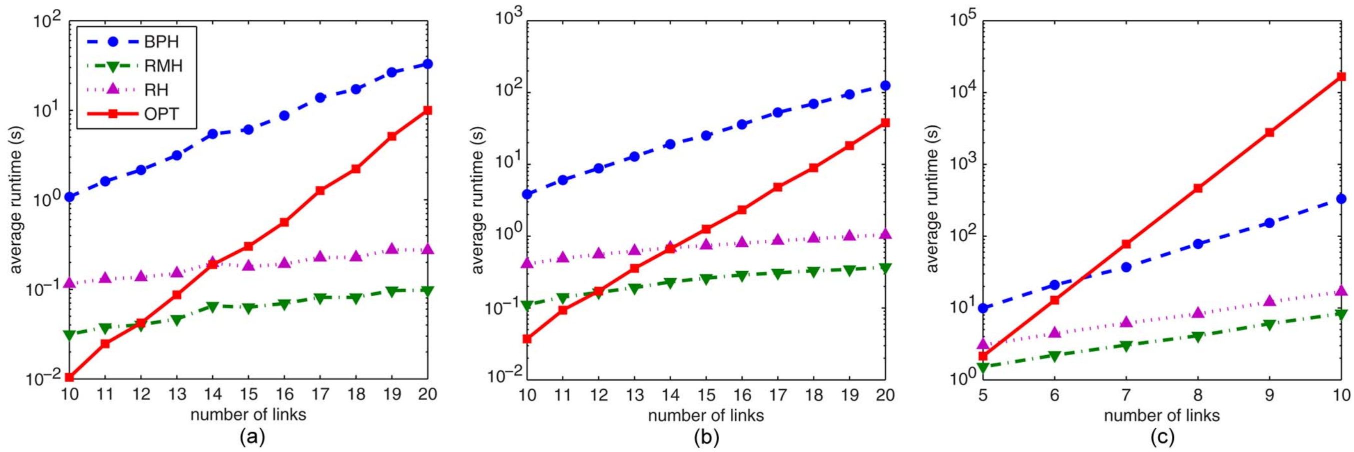

Fig. 4. Average runtime of BPH, RMH, RH, and OPT scheduling algorithms for a) constant rate, b) continuous rate, c) discrete rate transmission models.

negligible compared to the OPT algorithm for 30 links. Note that it is impossible to solve the OPT algorithm for higher than 20 links using MATLAB while the BPH algorithm provides solutions for larger number of the links. The reason for this intractability difference is that the OPT algorithm solves the scheduling problem as a pure IP problem whereas the BPH algorithm searches for the optimal solution over a BPH tree by exploiting the fathoming and the efficiency of the CGM based method together with the proposed heuristic method to solve the pricing problem. Furthermore, the runtime behavior of the scheduling algorithms for the discrete rate transmission model is very different from that of constant and continuous rate models. The average runtime of the OPT algorithm increases dramatically as the number of links increases and becomes 100 times greater than that of the BPH algorithm for 10 links. This is a direct result of the exponential complexity of the FTFA power control algorithm: The OPT algorithm uses FTFA to determine the time slot length for all possible subsets of the links whereas the BPH algorithm uses FTFA for smaller number of links since the average number of concurrently transmitting links in the optimal solution is smaller than the number of the links in the network.

Fig. 5 shows the length of the schedule for constant rate, continuous rate and discrete rate transmission models normalized by that of the continuous rate model. As expected, the length of the schedule is lowest and highest for continuous rate and constant rate models respectively. The schedule lengths of the discrete rate and constant rate transmission models get closer to that of the continuous rate model as the number of the links increases mainly due to exponentially increasing number of possible link rate vectors in the number of the links leading to the selection of a better link rate vector.

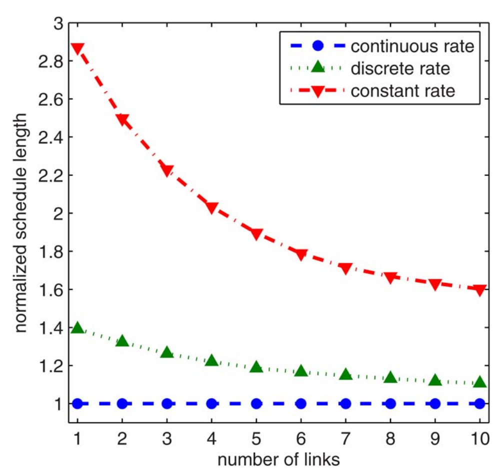

Fig. 5. Effect of transmission model on the schedule length.

IX. Conclusion

We formulate the joint optimization of power control, rate adaptation and scheduling with the goal of generating minimum length schedule given traffic demands on the links in practical packet based wireless *ad hoc* networks. In contrast to the traditional approach where the optimization problem ignores the packet level details assigning the transmission of any amount of data to the time slots at different rates, we assume that only an integer number of packets can be transmitted in a time slot. We first formulate the optimal power control and rate adaptation problem with the goal of minimizing the length of the time slot given the number of packets transmitted by the subset of the links and propose both optimal and fast heuristic algorithms. Given the minimum time slot lengths corresponding to all the subset of the links concurrently transmitting any possible number of packets, the joint optimization of power control, rate adaptation and scheduling problem can then be reduced to a pure scheduling problem. The scheduling problem is an IP problem in which the number of variables is exponential in the number of the links in the network. In order to solve this large-scale IP, we propose Branch and Price method and two Column Generation method based heuristic algorithms, and compare their performance to that of the Traditional Scheduling algorithm that ignores the constraints of the packet transmissions via extensive simulations. The proposed algorithms outperform the traditional scheduling algorithm. Moreover, Branch and Price method based heuristic algorithm (BPH) provides performance very close to the optimal solution and robust to the increasing number of the links with much smaller runtime than that of the optimal algorithm enabling the usage of the method in large networks. The Column Generation method based heuristic algorithms on the other hand provide performance slightly worse than the BPH but degrading more as the number of the links increases with runtime much smaller than the BPH.

In the future, we are planning to extend the proposed framework for different objectives such as throughput maximization and energy minimization to provide realistic solutions for various packet based wireless network applications.

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