Fast Scheduling for Delay Minimization in UWB Wireless Networks

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***Abstract*—We study the optimal scheduling problem for delay minimization subject to traffic demand, transmit power and Signal-to-Noise-plus-Interference Ratio (SNIR) constraints in rate-controlled Ultra-Wideband (UWB) wireless networks. We first formulate the Linear Programming (LP) problem where the number of variables is exponential in the number of the links. We then propose the heuristic algorithm called Exclusion Region and Utility Maximization based Column Generation Method (EXUM-CGM) to solve the problem rapidly and efficiently. In EXUM-CGM, we decompose the large scale problem into two sub-problems, Restricted Master Problem (RMP) and Pricing Problem (PP). We adapt the exclusion region concept commonly used in UWB systems to the initialization of the RMP. Since the PP formulation is a non-linear integer programming problem, we propose a heuristic algorithm based on utility maximization. Through the simulations, we show that EXUM-CGM decreases the runtime of the exponential LP problem significantly while achieving very close-to-optimal solutions.**

***Index Terms*—Scheduling, delay minimization, UWB.**

I. INTRODUCTION

IN this paper, we focus on the time-critical applications of UWB where the objective is to minimize the length of the schedule, i.e. delay minimization, given the traffic demand of the links in the network in contrast to the commonly studied multimedia applications where the aim is to maximize the throughput while providing a certain level of fairness to the links [1], [2].

Determining the best sets of links to be concurrently active to constitute the optimal schedule for delay minimization requires solving very large scale problems exponential in the number of the links. Column Generation Method (CGM), which is an elegant and rapid method used for solving such large scale problems [3], has been previously considered to decompose this problem and generate rapid and efficient heuristic algorithms for power-controlled wireless networks where each link transmits at a fixed rate no matter what the concurrently scheduled links are [4]. For UWB wireless networks on the other hand it has been shown that the sender can adapt its transmission rate to the SNIR level to meet the bit error rate requirement by changing the coding rate or modulation scheme easily [1], [2]. Hence, the heuristics presented in [4] for fixed-rate scheduling cannot be used for UWB wireless networks. In fact, in the existence of SNIR requirements on the links, fixed-rate allocation based problem formulation is an instance of rate-adaptation based problem formulation where the rate variables are eliminated by assigning them fixed predetermined values.

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In this paper, we study the optimal scheduling problem for delay minimization in rate-adaptive UWB wireless networks.

II. SCHEDULING FOR DELAY MINIMIZATION

A. Optimal Scheduling, Power and Rate Allocation Problem

The optimal scheduling, rate adaptation and power control problem for the objective of delay minimization subject to traffic demand, transmit power and SNIR constraints in UWB wireless networks is formulated as follows:

minimize

$$\sum_{n=1}^{N} t^{(n)} \tag{1}$$

subject to

$$\sum_{n=1}^{N} t^{(n)} x_l^{(n)} \geq R_l, \quad l \in [1, L] \tag{2}$$

$$p_l^{(n)} \leqslant p_{max}, \quad l \in [1, L], n \in [1, N] \tag{3}$$

$$x_l^{(n)} \leq K \frac{p_l^{(n)} h_{ll}}{\beta_l \left(N_0 + \sum_{k=1, k \neq l}^{L} p_k^{(n)} h_{kl} \gamma \right)}, \quad l \in [1, L], n \in [1, N] \tag{4}$$

$$a_{lk} + 1_{\left\{p_l^{(n)} > 0\right\}} + 1_{\left\{p_k^{(n)} > 0\right\}} \leq 2, \quad l, k \in [1, L], n \in [1, N] \tag{5}$$

variables

$$t^{(n)} \geq 0, p_l^{(n)} \geq 0, x_l^{(n)} \geq 0, \quad l \in [1, L], n \in [1, N] \tag{6}$$

where L is the number of links, N is the number of time slots, R_l is the data requirement of link l, p_{max} is the maximum allowed power determined by the UWB regulations, K is a system constant mapping SNIR to the achievable rate, N_0 is the background noise energy plus interference from non-UWB systems, h_{ll} is the power gain of link l, h_{kl} is the power gain from the transmitter of link k to the receiver of link l, γ is a parameter depending on the shape and repetition time of the UWB pulse, β_l is the threshold for the ratio of SNIR to the rate for link l determined based on various considerations such as desired bit error rate and modulation schemes, and a_{lk} is a constant that takes the value 1 if links l and k share a common node and 0 otherwise. The variables of the optimization problem are $t^{(n)}$, the length of the n-th time slot; $x_l^{(n)}$, the transmission rate of link l in time slot n; $p_l^{(n)}$, the transmit power of link l in time slot n.

Equation (2) represents the data constraint of the links in the network. Equation (3) states the upper bound for the transmit power of the links. Equation (4) states the upper bound on the transmission rate of the links [5]. This formulation for the maximum achievable transmission rate is based on the UWB characteristics which are adaptability of the transmission rate to the SNIR level at the receiver and linear relation between transmission rate and SNIR level due to very large bandwidth. SNIR calculation is valid for both pulse based time hopping and direct sequence UWB communications and independent of the channel model, i.e. the power gains h_{lk} can be calculated by using the UWB channel model of any application [1], [5], [6]. Finally, Equation (5) states that any node in the network cannot transmit to and receive from more than one node simultaneously.

B. Optimal Scheduling Problem

Given the power allocation of the links in a time slot, the achievable rate region for a link given in Equation (4) is independent of the rates of the concurrently transmitting links. Since higher rate results in smaller transmission delay, the optimal transmission rate is the maximum achievable rate for delay minimization. Moreover, it has been shown in [7] that any feasible average rate is achievable by $0/p_{max}$ power allocation to the links meaning that any feasible delay of a network can be achieved by $0/p_{max}$ power allocation so no power control is needed for delay minimization in UWB wireless networks. The scheduling problem is therefore independent of the power and rate allocation allowing us to reduce the joint formulation given in Section II-A to a pure scheduling problem.

Let $\mathcal{E} = \{\mathcal{E}_k : 1 \leq k \leq |\mathcal{E}|\}$ denote the set of all possible subsets of the link set $\mathcal{L} = \{1, 2, ..., L\}$ that can transmit concurrently considering the constraint given in Equation (5). Note that $|\mathcal{E}| \leq 2^L$ with equality for the case where no two links share any common node.

Let X denote an $L \times |\mathcal{E}|$ matrix such that the element in the l-th row and k-th column of X denoted by $x_l^{(k)}$ is the optimal transmission rate of link l if link l is in the subset $\mathcal{E}_k$ and 0 otherwise.

The optimal scheduling problem for delay minimization is then formulated as a Linear Programming (LP) problem as follows:

minimize

$$\mathbf{1}^T t \tag{7}$$

subject to

$$Xt \geq R \tag{8}$$

variables

$$t \geq \mathbf{0} \tag{9}$$

where $\mathbf{1}$ is $|\mathcal{E}| \times 1$ all-ones vector, $\mathbf{0}$ is $|\mathcal{E}| \times 1$ all-zeros vector, R is the vector containing the data requirements of the links, i.e. $R = [R_1, R_2, ..., R_L]$. The variable of the LP problem is the vector $t = [t^{(1)}, t^{(2)}, ..., t^{(|\mathcal{E}|)}]$, where $t^{(k)}$ is the time slot length allocated to the subset $\mathcal{E}_k$. Equation (8) represents the data requirements of the links in the network.

It requires an exponential effort to both form the X matrix and solve this LP problem since the number of variables is exponential in the number of the links. This approach is intractable for large number of links even with the use of efficient optimization problem solvers such as Fast Lipschitz [8]. On the other hand, by virtue of Caratheodory theorem, the number of non-zero variables in the optimal solution of this LP problem is at most L suggesting the use of the column generation method where the columns of X are generated on-the-fly iteratively as needed.

C. Exclusive Region and Utility Maximization Based Column Generation Method (EXUM-CGM)

The Column Generation Method (CGM) decomposes large scale original LP problem into two subproblems, a Restricted Master Problem (RMP) and a Pricing Problem (PP), and solves them iteratively.

1) Restricted Master Problem: The Restricted Master Problem is similar to the original problem except that only a small subset of feasible link sets $\mathcal{E}^s \subset \mathcal{E}$ is considered:

minimize

$$\mathbf{1}^T t \tag{10}$$

subject to

$$X^s t \geq R \tag{11}$$

variables

$$t \geq \mathbf{0} \tag{12}$$

where X^s denotes the transmission rate matrix corresponding to the subset $\mathcal{E}^s$ and the length of all-ones vector $\mathbf{1}$, all-zeros vector $\mathbf{0}$ and variable t is $|\mathcal{E}^s| \times 1$.

To start CGM, we need an initial X^s matrix that guarantees the feasibility of the RMP. An intelligent choice of X^s can reduce the number of iterations required in CGM since the solution converges to the optimal solution in each iteration of CGM and the initial X^s matrix determines the distance of the initial solution to the optimal point. We therefore propose the heuristic algorithm called Exclusion Region Based Algorithm (ERBA) based on the exclusion region concept commonly used in UWB network scheduling algorithms to obtain a good initial subset $\mathcal{E}^s$. The algorithm determines the first subset in $\mathcal{E}^s$ by choosing the maximal subset of nodes in $\mathcal{L}$ that are outside the exclusion region of each other then proceed by removing the link that satisfies its data requirement and including maximal subset of additional links satisfying exclusion region constraint to determine the next subset to be included in $\mathcal{E}^s$ in each step.

Algorithm 1 Exclusion Region Based Algorithm

$\mathcal{E}^s = \emptyset$; $\mathcal{E}_k = \emptyset$; $D = \mathcal{L}$; $k = 1$;
while $D \neq \emptyset$ **do**
  while there are links in D outside the exclusion regions of the links in $\mathcal{E}_k$ **do**
    add an arbitrary link outside the exclusion regions of the links in $\mathcal{E}_k$;
  end while
  $\mathcal{E}^s = \mathcal{E}^s + \mathcal{E}_k$;
  drop the link that satisfies its data requirement from D and $\mathcal{E}_k$;
  $\mathcal{E}_{k+1} = \mathcal{E}_k$; $k = k + 1$;
end while

We can solve RMP and its dual problem with simplex method in polynomial time to obtain its primal optimal solution t^p and dual optimal solution t^d respectively. Since we consider only an arbitrary subset $\mathcal{E}^s \subset \mathcal{E}$, t^p is not necessarily the optimal solution to the original problem. In the original problem, the cost coefficient of every variable in the objective is 1. Then, the reduced cost of a column $x^{(k)}$ in the matrix X but not in X^s is

$$c^{(k)} = 1 - \left(t^d\right)^T x^{(k)} \tag{13}$$

If the reduced cost $c^{(k)}$ of any column $x^{(k)}$ is less than 0, the objective function of the RMP can be further reduced by adding the column $x^{(k)}$ to the matrix X^s. Otherwise, the current solution t^p is the optimal solution to the original problem. Instead of calculating the reduced cost of each column $x^{(k)}$ in the matrix X but not in X^s, which requires an exponential effort, we can solve the following Pricing Problem which will generate the column with the minimum reduced cost among all columns in X matrix.

2) Pricing Problem: Pricing Problem is formulated as follows:

maximize

$$\sum_{l=1}^{L} t_l^d K \frac{b_l p_{max} h_{ll}}{\beta_l \left(N_0 + \sum_{k \neq l} b_k p_{max} h_{kl} \gamma \right)} \tag{14}$$

subject to

$$a_{lk} + b_l + b_k \leq 2, \quad l,k \in [1, L] \tag{15}$$

variables

$$b_l \in \{0, 1\}, \quad l \in [1, L] \tag{16}$$

where t_l^d is the l-th element of the vector t^d representing the dual optimal solution of the RMP in the current iteration. The variable of the optimization problem is b_l, which takes value 1 if link l is active and 0 otherwise.

The objective of the optimization problem is to minimize the reduced cost given in Equation (13) so maximize $\left(t^d\right)^T x^{(k)}$ over all columns $x^{(k)}$ of the matrix X leading to maximizing $\sum_{l=1}^{L} t_l^d x_l$ over all possible rate columns $[x_1, x_2, ..., x_L]$ in the matrix X. Once the active links are determined through variable b_l for each link l, the rate used in the corresponding column of the matrix X is the maximum achievable rate given in Equation (4) with the assigned power allocation p_{max} to the active links as explained in Section II-B, i.e. $x_l = K \frac{b_l p_{max} h_{ll}}{\beta_l \left(N_0 + \sum_{k \neq l} b_k p_{max} h_{kl} \gamma \right)}$, resulting in the objective function given in Equation (14). Equation (15) represents the constraint on the concurrent transmissions previously given in Equation (5) in terms of integer variables b_l.

The PP formulation is a non-linear integer programming problem for which there is no known polynomial-time algorithm. We therefore propose the following heuristic algorithm called Utility Maximization Algorithm (UMA) to efficiently and rapidly solve PP. We define the utility u_S for a set S of links as

$$u_S = \sum_{l \in S} t_l^d K \frac{p_{max} h_{ll}}{\beta_l \left(N_0 + \sum_{k \neq l, k \in S} p_{max} h_{kl} \gamma \right)} \tag{17}$$

UMA algorithm iteratively adds links to the initially empty set G such that in each iteration, the link picked is the one that maximizes the utility of the set after its addition. The algorithm stops when the set G includes all the links in the link set $\mathcal{L}$ or the addition of a link cannot increase the utility of the set G. If the utility of the resulting set G returned by the UMA algorithm is higher than 1, the corresponding rate vector is passed to the RMP. Otherwise, the CGM algorithm stops.

Algorithm 2 Utility Maximization Algorithm

$G = \emptyset$; $D = \mathcal{L}$;
while $G \neq \mathcal{L}$ **do**
  if $\max_{i \in D} u_{G+i} > u_G$ **then**
    $k = \arg\max_{i \in D} u_{G+i}$;
    $G = G + k$; $D = D - k$;
  else
    break;
  end if
end while

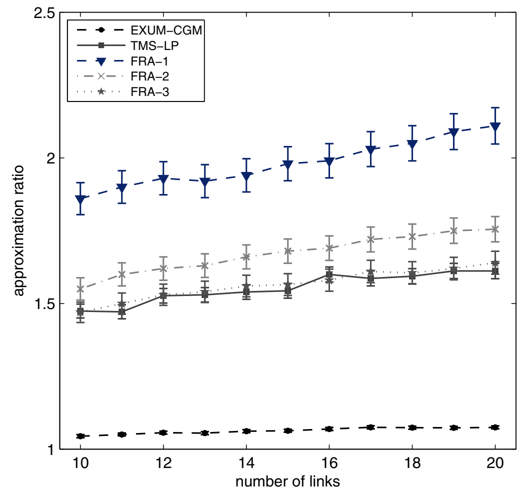

Fig. 1. Approximation ratio of EXUM-CGM, TMS-LP and FRA algorithms for different number of links.

III. Performance Evaluation

We used MATLAB on a computer with a 2.5GHz CPU and 4GB RAM to run the simulations. The attenuation of the links are determined using the path loss model given by $PL(d) = PL(d_0) - 10\alpha \log_{10}\left(\frac{d}{d_0}\right) + Z$, where d is the distance between the transmitter and receiver, d_0 is the reference distance, $PL(d_0)$ is the path loss at the reference distance, α is the path loss exponent and Z is a Gaussian random variable with zero mean and σ_z^2 variance. The parameters of the model used in the simulations are $\alpha = 4$, $\sigma_z^2 = 2$, $PL(d_0) = 30dB$, $d_0 = 1m$, $N_0 = 10^{-8}W/Hz$, $p_{max} = 10mW$, $\gamma = 10^{-3}$, $K = 10^6$, $\beta_l = 10dB$ for all $l \in \mathcal{L}$. Simulation results are obtained based on 1000 independent random network topologies where various numbers of links with $1m$ fixed length are uniformly distributed over square areas of different sizes indicating different network densities. The reason for choosing fixed length links is to perform a worst case performance comparison with alternative scheduling algorithms.

Fig. 1 shows 95% confidence interval bars depicted around the mean of the approximation ratio of EXUM-CGM, Throughput Maximization Set based LP (TMS-LP) and Fixed Rate Allocation (FRA) algorithms for different number of links. The approximation ratio is the ratio of the delay obtained by the heuristic algorithm to the delay obtained by the optimal exponential LP formulation (EXP-LP). In TMS-LP, first the links are assigned to a number of time slots proportional to their traffic weights such that a throughput maximizing subset of links is allocated to each

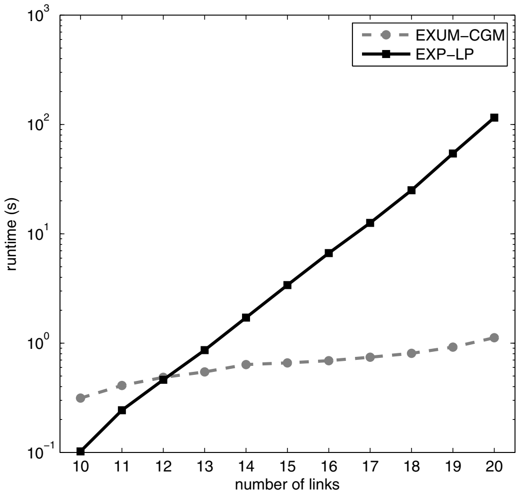

Fig. 2. Average runtime of EXUM-CGM and EXP-LP for different number of links.

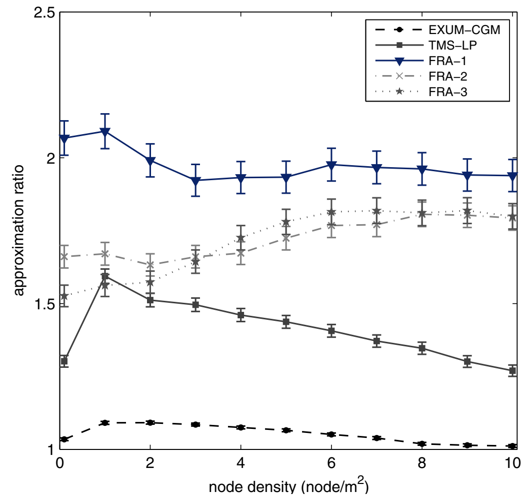

Fig. 3. Approximation ratio of EXUM-CGM, TMS-LP and FRA algorithms for different network densities.

time slot using the exclusion region concept in [1] then the resulting matrix X^s containing only these predetermined throughput maximizing subsets is input to the LP problem given in Equations (10) and (11) to determine the schedule. In FRA, first all subsets of the links feasible at a predetermined fixed transmission rate are determined then the resulting rate matrix X each column containing the fixed rate in all active links in each feasible subset is input to the LP problem given in Equations (7) and (8) to determine the schedule. We simulated FRA algorithm for three predetermined rate allocations considering each network separately. Let x_1 and x_2 be the minimum and maximum fixed transmission rate to get a feasible schedule using FRA algorithm. In FRA-1, FRA-2 and FRA-3, fixed transmission rates are calculated as $(2x_1 + x_2)/3$, $(x_1 + x_2)/2$ and $(x_1 + 2x_2)/3$ respectively. The approximation ratio of EXUM-CGM is closer to 1 with a smaller variance than the other algorithms degrading only slightly as the number of links increases. On the other hand, the approximation ratio of TMS-LP algorithm is around 1.5 with higher variance than EXUM-CGM and increases significantly as the number of links increases demonstrating the unsuitability of throughput maximization based identification of the concurrently transmitting subsets for delay minimization problem. The approximation ratios of FRA algorithms are even worse than TMS-LP with higher variances demonstrating the superiority of adaptive rate over fixed rate allocation.

Fig. 2 compares the average runtime of EXUM-CGM and EXP-LP. The runtime of EXUM-CGM increases linearly in the number of the links compared to the exponential increase of that of EXP-LP. Fig. 3 on the other hand shows the robustness of EXUM-CGM to varying network density. For a wide range of network density, EXUM-CGM performs close to optimal and again superior than the other algorithms.

IV. Conclusion and Discussion

In this paper, we first formulate the optimal scheduling problem for delay minimization subject to traffic demand, transmit power and SNIR constraints in rate-adaptive UWB wireless networks as an LP problem where the number of variables is exponential in the number of the links. We then propose the heuristic algorithm called Exclusion Region and Utility Maximization based Column Generation Method (EXUM-CGM). EXUM-CGM decomposes the large scale LP problem into Restricted Master Problem (RMP) and Pricing Problem (PP) by adapting the elegant Column Generation Method (CGM) used for large scale problems. Simulations show that EXUM-CGM performs very close to the optimal and its average runtime is much smaller than that of the optimal exponential LP over many different random topologies for varying number of links and node densities. For the future work, we will adapt CGM for solving the problems of delay minimization with energy constraints and energy minimization with delay constraints, and propose a general framework for the application of CGM to any wireless network scheduling problem with any constraint depending on the specific application requirements.

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