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Joint Optimization of Wireless Network Energy Consumption and Control System Performance in Wireless Networked Control Systems

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***Abstract*—Communication system design for Wireless Networked Control Systems (WNCSs) requires satisfying the high reliability and strict delay constraints of control systems for guaranteed stability, with the limited battery resources of sensor nodes, despite the wireless networking induced non-idealities. These include non-zero packet error probability caused by the unreliability of wireless transmissions and non-zero delay resulting from packet transmission and shared wireless medium. In this paper, we study the joint optimization of control and communication systems incorporating their efficient abstractions practically used in real-world scenarios. The proposed framework allows including any non-decreasing function of the power consumption of the nodes as the objective, any modulation scheme and any scheduling algorithm. We first introduce an exact solution method based on the analysis of the optimality conditions and smart enumeration techniques. Then, we propose two polynomial-time heuristic algorithms based on intelligent search space reduction and smart searching techniques. Extensive simulations demonstrate that the proposed algorithms perform very close to optimal and much better than previous algorithms at much smaller runtime for various scenarios.**

***Index Terms*—wireless networked control system, optimization, wireless communication, control system, energy, delay, reliability, scheduling, stability.**

I. Introduction

WNCSs are spatially distributed control systems in which sensors, actuators and controllers communicate through a wireless network [2]. The usage of wireless communication in control systems results in low cost and flexible network architectures by decreasing the cost of the installation, modification and upgrade of the system components compared to their wired equivalent. WNCSs have therefore been finding various applications in industrial automation [3], building automation [4], automated highway [5] and smart grid [6] with standardization efforts of industrial organizations such as International Society of Automation (ISA) [7] and Highway Addressable Remote Transducer (HART) [8].

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The communication system design for a WNCS requires guaranteeing the performance and stability of control system, with the limited battery resources of sensor nodes, despite the unreliability of wireless transmissions and delay resulting from packet transmission and shared wireless medium. The key parameters that need to be considered by both control and communication systems are the packet error probability, delay requirement and sampling period of the sensor nodes in the network. Decreasing the values of these parameters improves the performance of the control system. However, the energy consumed in the wireless transmission of the sensor nodes is a monotonically decreasing function of these parameters, when they are formulated as a function of the transmission power and rate of the sensor nodes in the network. Some of the works in the literature focused on the design of optimal controllers given the delay and packet loss of wireless communication systems [9], [10], whereas others studied the design of the scheduling of wireless communication systems given the packet loss of the wireless links, and the delay requirement and sampling period of sensor nodes satisfying a certain control system performance [11], [12], [13], [14]. The optimal performance of WNCSs, considering the trade-off between communication and controller performance, however can only be achieved by jointly optimizing the control and communication systems, which has received little attention in the literature mainly due to the lack of efficient abstractions of both systems. Such optimization requires modeling the interaction between control and wireless communication subsystems through their efficient and accurate abstractions, considering real-world scenarios; and considering all the communication system parameters including transmission power, rate and scheduling of sensor nodes, and the control system parameters including sampling period of sensor nodes; without sacrificing problem tractability.

The joint optimization of control and communication systems has been studied for Networked Control Systems (NCS) [15], [16], [17], [18]. Assuming no packet error occurs unless there is a collision in the wired network, [15], [16] investigate the optimization of scheduling subject to the sampling period and delay requirements of the sensor nodes, while [17], [18] focus on the optimization of the sampling period and delay parameters of the sensors with the objective of minimizing the overall performance loss while ensuring schedulability. The formulations and proposed solutions in these studies, however, cannot be applied to WNCS due to the requirement of including the non-zero packet error probability of wireless

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transmission and its dependence on the transmission power and rate of sensor nodes.

The optimization problem formulations for WNCSs mainly aim to address the trade-off between the energy consumption of the wireless communication and the performance of the control system [19], [20], [21], [22]. In [19], [20], the energy consumption of sensor nodes in the wireless network is minimized subject to the stability and performance requirements of the control system, whereas [21], [22] maximize the control system performance subject to the packet loss probability of wireless links and/or the energy constraints of sensor nodes. However, these studies mostly assume a constant packet loss probability over wireless links and fixed energy consumption per packet transmission without analyzing their dependency on the transmission power and rate of the sensor nodes, and the scheduling of sensor node transmissions.

The joint optimization of controller and communication systems considering all the wireless network induced imperfections including packet error and delay, and all the parameters of both wireless communication and control systems has been recently studied in [23]. However, the optimization framework and therefore the solutions are limited to the objective of minimizing the total power consumption of the communication system, M-ary Quadrature Amplitude Modulation (MQAM) as the modulation scheme and Earliest Deadline First (EDF) as the scheduling algorithm. The goal of this paper is to extend this study by generalizing the optimization framework incorporating a generalized objective function and removing the reliance on a specific modulation scheme and a scheduling algorithm, and propose solution methods and algorithms that can be applicable to a wide range of control applications.

The original contributions of the paper are listed as follows:

- We provide a generalized framework for the joint optimization of controller and communication systems incorporating their efficient abstractions, mainly derived from the usage in practical scenarios. The framework encompasses any non-decreasing function of the power consumption of the nodes in the objective, any modulation scheme and scheduling algorithm. This optimization framework is expected to lead to broader adoption in many real-world control applications.
- Upon analyzing the optimality conditions on the variables of the generalized optimization problem, we propose an optimal algorithm to solve the problem in reasonable time based on smart enumeration techniques.
- We propose two polynomial-time heuristic algorithms based on a search space reduction technique that exploits the utilization concept used in real-time scheduling, energy consumption dominance relations of the constellation size of each sensor node and smart searching technique that proceeds by evaluating the feasibility conditions and objective function of neighboring constellation size vectors. These search space reduction technique based heuristic algorithms decrease the complexity of the optimal algorithm significantly while keeping the performance very close to optimal.
- We illustrate the superiority of the proposed heuristic algorithms to previously proposed solution methods in terms of both closeness to the optimality and average runtime for various network sizes, modulation schemes, objective functions, and control system parameters via extensive simulations.

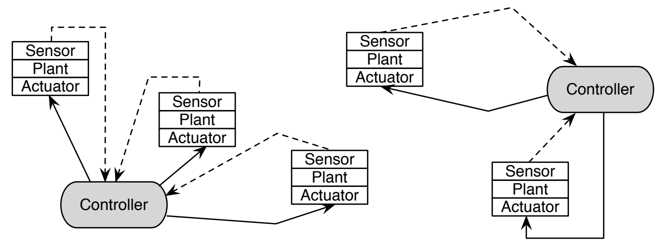

Fig. 1: Overview of the WNCS architecture.

The rest of the paper is organized as follows. Section II describes the system model and assumptions used throughout the paper. The generalized joint optimization of controller and communication systems has been formulated as a non-convex Mixed Integer Programming problem and reduced to Integer Programming (IP) problem based on the analysis of the optimality conditions in Section III. Section IV presents an optimal smart enumeration based algorithm. Section V provides polynomial-time heuristic solution methods employing utilization based search space reduction and smart searching techniques. Simulation results are presented in Section VI. Finally, concluding remarks are given in Section VII.

II. System Model and Assumptions

The system model and assumptions are detailed as follows.

1) The architecture of a WNCS is depicted in Fig. 1. Multiple plants are controlled over a wireless communication network. A plant is a physical component of a WNCS. Sensor nodes attached to the plants sample their outputs periodically and then transmit to the controller commanding that particular plant through wireless channel, which induces nonzero transmission delays and packet errors. Upon successful reception of the sensor data, the controller computes a new control command to maintain the stability and high-performance operation of the control plant. The control command is then forwarded to the actuator attached to the plant. We assume that actuators successfully receive commands since they are collocated with the controllers, as in heat, ventilation and air conditioning control systems, due to their high criticality [24].

   WNCS consists of multiple controllers, each controlling a certain physical domain of the control system. One of the controllers is assigned as the coordinator. Coordinator controller is responsible for time synchronization in the network, resource allocation for the network elements; i.e. running the resource allocation algorithms and informing the nodes about the decisions in a centralized framework, and monitoring the network topology and channel conditions.
2) The periodic information transfer between a sensor attached to a plant and the controller commanding that

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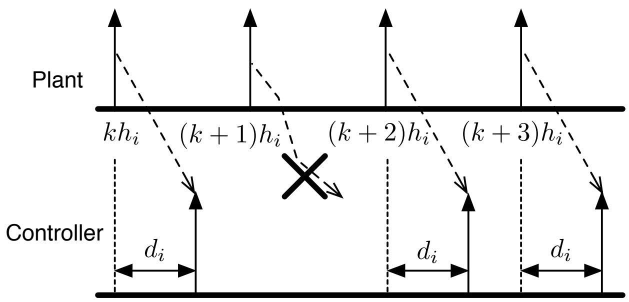

Fig. 2: Timing diagram of the wireless communication of a sensor and a controller.

plant is illustrated in Fig. 2. The sampling period, transmission delay and packet error probability of sensor node i are denoted by h_i, d_i, and p_i, respectively. We assume that $d_i \leq h_i$ since the packets are outdated and replaced with the new sampled data for a transmission delay beyond d_i. The retransmission of the outdated or lost packets, due to large transmission delay and packet errors respectively, are generally not useful for the control system since the latest information of the plant state is the most critical information for control applications. The packet error model is assumed to be a Bernoulli random process with probability p_i for node i for simplification. This assumption is valid when channel coherence time is less than the sampling period of the sensor nodes; e.g., fast fading channel environments and control applications with relatively large data sampling periods [25].

3) Time Division Multiple Access (TDMA) is considered as medium access control (MAC) protocol. Explicit scheduling of the node transmissions in TDMA allows meeting the strict delay and reliability requirement of control systems while minimizing the energy consumption of the sensor nodes by putting their radio in sleep mode when they are not scheduled to transmit or receive any packet. Moreover, mostly predetermined topology and data generation patterns of the sensor nodes in a WNCS decreases the synchronization and topology learning overhead associated with TDMA. TDMA is commonly used in industrial control applications [7], [8].
4) The time is divided into scheduling frames of fixed length, each of which is further partitioned into a beacon and variable number of variable-length time slots. Coordinator controller transmits the beacon periodically to maintain synchronization among the elements of the WNCS. Besides, in case of any change in the resource allocation and scheduling decisions upon variations of the channel conditions or network topology, the beacon additionally includes the updated schedule, and the transmission power, rate and sampling period of sensor nodes.
5) We assume a multi-modal operation for the sensor nodes such that they operate in sleep mode if they are not scheduled to transmit or receive a packet, in active mode if they are scheduled to transmit or receive a packet, and in transient mode while switching from active mode to sleep mode and vice versa. However, we consider only the power consumption in the transmission of the packets in the optimization problem since it is much larger than those in the sleep and transient modes, and the energy consumed in the reception of beacon packets is fixed [26], [27].
6) The performance and stability conditions for the WNCS have been formulated in the form of Stochastic Maximum Allowable Transfer Interval (MATI), defined as keeping the time interval between subsequent state vector reports from the sensor nodes to the controller below MATI value with a predefined probability, and Maximum Allowable Delay (MAD), defined as the maximum allowed packet delay for the transmission from the sensor node to the controller. Stochastic MATI and MAD constraints are efficient abstractions of the performance of control systems however have been considered only recently in the joint design with the communication systems [23].

   a) Stochastic MATI constraint is formulated as
$$\Pr\left[\mu_i(h_i, d_i, p_i) \leq \Omega\right] \geq \delta, \tag{1}$$
   where μ_i is the time interval between subsequent state vector reports of node i as a function of h_i, p_i and d_i; Ω is the MATI value; and δ is the minimum probability with which MATI requirement should be achieved. The values of Ω and δ are determined by the control application. For instance, in industrial automation, closed-loop machine controls have a stochastic MATI requirement with $\Omega = 100$ ms and $\delta = 0.999$ [7], [28]. Moreover, to allow IEEE 802.15.4 devices [29] to support a wide range of industrial applications, IEEE 802.15.4e standard [30] specifies an amendment to the IEEE 802.15.4 standard to enhance its latency and reliability performances for industrial automation. They have specified $\Omega = 10$ ms and $\delta = 0.99$. The air transportation system requires $\Omega = 4.8$ s and $\delta = 0.95$ [25]. In addition, the cooperative vehicular safety applications requires $\Omega = 100$ ms and $\delta = 0.95$ [31].
   The number of reception opportunities of the state vector reports is equal to $\left\lfloor \frac{\Omega}{h_i} \right\rfloor$ within each time interval of length Ω. Based on the assumption above on the modeling of packet error as a Bernoulli random process with probability p_i for node i, Eq. (1) can be rewritten as $1 - p_i^{\left\lfloor \frac{\Omega}{h_i} \right\rfloor} \geq \delta$.
   b) MAD constraint is formulated as
$$d_i \leq \Delta, \tag{2}$$
   where Δ is the MAD value to stabilize the control system. Typical Δ values are on the order of a few tens of milliseconds for fast control applica-

tions [7], [28], [32].

7) The average power consumption of sensor node i is formulated as a function of its sampling period, transmission delay and packet error probability as
$$W_i\left(h_i, d_i(b_i), p_i\right) = \frac{\left(W_i^t\left(b_i, p_i\right) + W_i^c\right) d_i(b_i)}{h_i}, \quad (3)$$
where b_i is the number of bits used per symbol or the constellation size, d_i is represented as a function of b_i for a predetermined modulation scheme, W_i^t is the transmission power calculated as a function of the parameters b_i and p_i for a given modulation, and W_i^c is the circuit power consumption in the active mode at the transmitter. In the following, we will use the notation of $W_i\left(h_i, b_i, p_i\right)$ instead of $W_i\left(h_i, d_i(b_i), p_i\right)$ for convenience. We assume that the average power consumption $W_i\left(h_i, b_i, p_i\right)$ and transmit power $W_i^t(b_i, p_i)$ satisfy the following properties:
   a) *$W_i\left(h_i, b_i, p_i\right)$ is a monotonically decreasing function of h_i.*
   b) *$W_i^t(b_i, p_i)$ is a monotonically decreasing function of p_i.*

   Property (a) follows from Eq. (3) and holds for any modulation scheme. Property (b) implies that a lower power consumption can be achieved at the expense of a higher packet error probability, keeping the remaining parameters fixed. It can be verified that these properties are satisfied in many modulation schemes including QAM and FSK (Frequency Shift Keying) [27].
8) We assume that the transmit power of a sensor node cannot exceed a maximum allowed power level $W^{t,\max}$ due to the limited weight and size of the sensor nodes. The maximum transmit power constraint is formulated as
$$W_i^t(b_i, p_i) \leq W^{t,\max}. \quad (4)$$
9) The schedulability constraint represents the feasibility of the allocation of the time slots corresponding to the given constellation size and sampling period of the nodes in the network, while the concurrent transmissions of the sensor nodes are not allowed, for a pre-determined scheduling algorithm. In other words, it represents whether a schedule can be constructed given the transmission duration and period of each node in the network under a pre-determined scheduling algorithm. The schedulability constraint is formulated as
$$\{(d_1(b_1), h_1), ..., (d_N(b_N), h_N)\} \in S^{feasible}, \quad (5)$$
where $S^{feasible}$ denotes the set of $\{(d_1(b_1), h_1), ..., (d_N(b_N), h_N)\}$ values with which a feasible schedule can be constructed. Any scheduling algorithm including EDF, Least Laxity First, Rate Monotonic and Deadline Monotonic scheduling algorithms [33] can be adopted in this framework. For instance, the schedulability constraint has been formulated as $\sum_{i=1}^{N} \frac{d_i}{h_i} \leq \beta$, where β is the utilization bound in the range $(0, 1]$, for pre-emptive EDF scheduling algorithm in [23].
10) We assume that the objective function $f(W_1(h_1, b_1, p_1), ..., W_N(h_N, b_N, p_N))$ is a non-decreasing function of $W_i(h_i, b_i, p_i)$ for all $i \in [1, N]$, where N is the number of nodes in the network. This assumption holds for many commonly used objective functions. Some examples can be listed as follows:
$$f(W_1(h_1, b_1, p_1), ..., W_N(h_N, b_N, p_N)) = \sum_{i\in[1,N]} W_i(h_i, b_i, p_i) \quad (6a)$$
$$f(W_1(h_1, b_1, p_1), ..., W_N(h_N, b_N, p_N)) = \sum_{i\in[1,N]} \log W_i(h_i, b_i, p_i) \quad (6b)$$

III. Joint Optimization of Control and Communication Systems

Efficient abstractions of control and communication systems given by stochastic MATI and MAD constraints in Eqs. (1)-(2), and maximum transmit power and schedulability constraints in Eqs. (4)-(5), respectively, enable investigating the interaction between the stability of the control system and power consumption of the wireless communication network. The formulations incorporate both control system parameter, sampling period h_i for node i, and communication system parameters, constellation size b_i and packet error probability p_i for node i. Given the modulation scheme, the transmission power and rate of a sensor node i can be represented as functions of b_i and p_i, as exemplified for MQAM modulation scheme in [23]. The transmission delay of a sensor node is then inversely proportional to its transmission rate. Decreasing the packet error probability, delay or sampling period improves the stability of the control system while increasing the power consumption of the network. The parametrization of the control and wireless communication systems through these parameters, therefore, allows formulating a joint optimization of control and communication systems addressing this trade-off.

The joint optimization problem aims to minimize the generalized non-decreasing function of the power consumption of the sensor nodes while satisfying the stochastic MATI and MAD constraints guaranteeing the stability of the control systems, and the maximum transmit power and schedulability constraints of the wireless communication network.

$$\min_{h_i, b_i, p_i, i\in[1,N]} \quad f(W_1(h_1, b_1, p_1), ..., W_N(h_N, b_N, p_N)) \quad (7a)$$
$$\text{s.t.} \quad \left\lfloor \frac{\Omega}{h_i} \right\rfloor \ln p_i - \ln\left(1 - \delta\right) \leq 0, \quad i \in [1, N], \quad (7b)$$
$$0 < d_i(b_i) \leq \min\left\{\Delta, h_i\right\}, \quad i \in [1, N], \quad (7c)$$
$$0 < h_i \leq \Omega, \quad i \in [1, N], \quad (7d)$$
$$0 < p_i < 1, \quad i \in [1, N], \quad (7e)$$
$$W_i^t(b_i, p_i) \leq W^{t,\max}, \quad i \in [1, N], \quad (7f)$$
$$\{(d_1(b_1), h_1), ..., (d_N(b_N), h_N)\} \in S^{feasible}. \quad (7g)$$

Eq. (7a) represents the generalized objective as a function of the power consumption of the nodes. Eqs. (7b) and (7c)

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represent the stochastic MATI and MAD constraints, respectively. Eq. (7d) states that the sampling period of the nodes must be less than or equal to the MATI. Eq. (7e) states the lower and upper bounds of the packet error probability. Eq. (7f) provides the maximum transmit power constraint. Finally, Eq. (7g) represents the schedulability constraint. The variables of the problem are $h_i, i \in [1, N]$, the sampling period of the nodes; $b_i, i \in [1, N]$, the constellation size of the nodes; and $p_i, i \in [1, N]$, the packet error probability of the nodes.

This optimization problem is a Mixed-Integer Programming problem in this current form thus difficult to solve for global optimum [34]. Therefore, we analyze the optimality relations among the optimization variables in order to convert the problem into a more tractable problem. Following lemma states the optimality conditions for the sampling period and the packet error probability of a sensor node.

Lemma 1: The optimal sampling period and packet error probability, denoted by h_i^* and p_i^* respectively, satisfy

$$\frac{\Omega}{h_i^*} = \frac{\ln(1-\delta)}{\ln p_i^*} = k_i, \tag{8}$$

where k_i is a positive integer for all $i \in [1, N]$.

Proof: We prove $\frac{\Omega}{h_i^*} = k_i$ by contradiction. Suppose that $\frac{\Omega}{h_i^*}$ is not a positive integer then $\left\lfloor \frac{\Omega}{h_i^*} \right\rfloor < \frac{\Omega}{h_i^*}$. If h_i^* increases such that the equality $\left\lfloor \frac{\Omega}{h_i^*} \right\rfloor = \frac{\Omega}{h_i^*}$ holds for the first time while satisfying the upper bound given in Eq. (7d), the stochastic MATI constraint given in Eq. (7b) still holds since the value of $\left\lfloor \frac{\Omega}{h_i^*} \right\rfloor$ does not change. The remaining constraints including h_i given in Eqs. (7c) and (7g) also still hold with this change. However, the objective cost function given in Eq. (7a) does not increase since it is a non-increasing function of h_i for each node $i \in [1, N]$. Similarly, we prove $\frac{\Omega}{h_i^*} = \frac{\ln(1-\delta)}{\ln p_i^*}$ by contradiction. Suppose that $\frac{\Omega}{h_{i^*}} < \frac{\ln(1-\delta)}{\ln p_i^*}$. If p_i^* increases such that the stochastic MATI constraint is satisfied with equality, the constraint given in Eq. (7f) still holds since the power consumption of node i is assumed to be a monotonically decreasing function of p_i. However, the objective cost function given in Eq. (7a) does not increase since it is a non-increasing function of p_i for each node $i \in [1, N]$. □

Lemma 1 allows the representation of optimization variables h_i and p_i in terms of a single variable k_i. Hence, we can eliminate them from the original optimization problem (7), which is then reformulated as

$$\min_{b_i, k_i, i \in [1,N]} \quad f(W_1(b_1, k_1), ..., W_N(b_N, k_N)) \tag{9a}$$

$$\text{s.t.} \quad 0 < d_i(b_i) \leq \min\left\{\Delta, \frac{\Omega}{k_i}\right\}, \quad i \in [1, N], \tag{9b}$$

$$W_i^t(b_i, k_i) \leq W^{t,\max}, \quad i \in [1, N], \tag{9c}$$

$$\left\{(d_1(b_1), \frac{\Omega}{k_1}), ..., (d_N(b_N), \frac{\Omega}{k_N})\right\} \in S^{feasible}, \tag{9d}$$

where $W_i(b_i, k_i)$ and $W_i^t(b_i, k_i)$ are obtained by replacing h_i and p_i by their expression of their optimal values as a function of k_i in $W_i(h_i, b_i, p_i)$ and $W_i^t(b_i, p_i)$, respectively, based on Lemma 1. The constraints given in Eqs. (9b), (9c) and (9d) correspond to those in Eqs. (7c), (7f) and (7g), respectively, and the remaining constraints in the optimization problem (7) are removed by exploiting the additional constraint of k_i being a positive integer.

We further proceed the optimality analysis with the following lemma stating the relation between the optimal values of k_i and b_i.

Lemma 2: The optimal value of k_i is the minimum positive integer satisfying Eq. (9c) and can be expressed as a function of b_i, denoted by $k_i^*(b_i)$.

Proof: Since the objective function is a non-increasing function of h_i and p_i, it is a non-decreasing function of k_i due to Lemma 1. Therefore, minimizing the objective function requires finding the minimum positive integer satisfying the constraints of the optimization problem given in Eqs. (9b), (9c) and (9d). Decreasing k_i does not shrink the feasibility regions for b_i determined by the constraints (9b) and (9d). Hence k_i^* is the minimum positive integer satisfying Eq. (9c) and can therefore be represented as a function of b_i given the transmit power function $W_i^t(b_i, k_i)$. □

Lemma 2 allows the representation of optimization variable k_i in terms of variable b_i in the optimization problem (9). We can also determine the minimum and maximum values of b_i for each sensor node i, denoted by $b_i^{\min}$ and $b_i^{\max}$, respectively, by evaluating the constraints given in Eqs. (9b) and (9c) based on Lemma 2. Then, the optimization problem can be further simplified as

$$\min_{b_i, i \in [1,N]} \quad f(W_1(b_1, k_1^*(b_1)), ..., W_N(b_N, k_N^*(b_N))) \tag{10a}$$

$$\text{s.t.} \quad b_i^{\min} \leq b_i \leq b_i^{\max}, \quad i \in [1, N], \tag{10b}$$

$$\left\{(d_1(b_1), \frac{\Omega}{k_1^*(b_1)}), ..., (d_N(b_N), \frac{\Omega}{k_N^*(b_N)})\right\} \in S^{feasible}. \tag{10c}$$

Since the constellation size b_i is integer for all $i \in [1, N]$, the optimization problem is an Integer Programming (IP) problem. Due to the non-convexity of the objective function in Eq. (10a) and the schedulability constraint in Eq. (10c), the relaxation of the problem is also non-convex in general, hence Branch and Bound techniques that are efficient to solve IPs are not applicable. However, enumeration techniques can be used to solve this IP problem optimally. We propose an optimal algorithm to solve the problem in reasonable runtime using smart enumeration techniques in Section IV and heuristic algorithms to achieve close-to-optimal solutions in polynomial runtime in Section V.

Before introducing the optimal and heuristic algorithms, in the following, we summarize the entire solution procedure for solving the joint optimization of control and communication systems as formulated by problem (7) and finding the optimal sampling period h_i, packet error probability p_i and constellation size b_i for each sensor node i:

1) Determine $b_i^{\min}$ and $b_i^{\max}$ values: The minimum and maximum values for b_i are determined for each sensor node i evaluating the constraints given in Eqs. (9b) and (9c), respectively, based on Lemma 2.
2) Determine b_i values: Using either the optimal algorithm

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presented in Section IV or one of the heuristic algorithms presented in Section V, b_i values are determined for each sensor node i.

3) Determine k_i values: Using b_i values obtained in step (2), k_i for each sensor node i can be obtained as the minimum positive integer value satisfying Eq. (9c), based on Lemma 2.
4) Determine h_i and p_i values: Using k_i values obtained in step (3), h_i and p_i values can be obtained using Eq. (8) stated by Lemma 1.

Upon determining the constellation size and packet error probability values, the transmission rate and power of the sensor nodes can be determined for a specific modulation scheme.

IV. Optimal Algorithm

The IP problem formulated in the previous section can be solved by an exhaustive search algorithm since the optimization variables are integer. Let $\mathbf{b}$ denote the constellation size vector where the i-th element of the vector, $\mathbf{b}_i$, is the constellation size of node $i \in [1, N]$. An exhaustive search algorithm simply calculates the objective value for each constellation size vector in the feasible region such that Eq. (10b) is satisfied for each sensor node $i \in [1, N]$, i.e. $b_i^{\min} \leq \mathbf{b}_i \leq b_i^{\max}$, and determines the one minimizing the objective function while satisying the schedulability constraint given by Eq. (10c). Such a search algorithm, however, is intractable for even medium network sizes. For example, for the number of nodes and the number of possible constellation size values for each sensor node $i \in [1, N]$ given by $N = 10$ and $A_i = b_i^{\max} - b_i^{\min} + 1 = 10$, respectively, 10^{10} possible constellation size vectors need to be checked for schedulability and value of objective function.

In the following, we present the proposed Optimal Fast Enumeration (OFE) Algorithm, which employs smart enumeration techniques to overcome this intractability issue, with the main characteristics listed as

1) ordering the set of constellation sizes in increasing power consumption for each node,
2) starting the evaluation of the schedulability and objective function with the constellation size vector corresponding to the minimum power consumption of each node,
3) pruning the schedulable constellation size vectors and the vectors with worse objective function value than the best vector so far,
4) regenerating the constellation size vectors for evaluation without repetitions covering all vectors in the case of no pruning by associating each vector with a number denoting the number of vectors it is branched into.

OFE Algorithm given by Algorithm 1 is described in detail as follows. The inputs of the algorithm are $A_i = b_i^{\max} - b_i^{\min} + 1$ possible constellation size values for each sensor node $i \in [1, N]$ resulting from Eq. (10b) in the simplified IP problem given in the previous section. Let b_{ij} denote the constellation size corresponding to the j-th minimum power consumption for node i. Let $deg(\mathbf{b})$ denote the degree of $\mathbf{b}$, which is defined as the number of vectors that vector $\mathbf{b}$ is branched into. The

Algorithm 1 **Optimal Fast Enumeration (OFE) Algorithm**

Input: $b_{ij},\ i \in [1, N],\ j \in [1, A_i]$;
Output: $\mathbf{b}^*, f^*$;

1: $f^* = \infty$;
2: $\mathbf{b} = (b_{11}, b_{21}, ..., b_{N1})$;
3: $deg(\mathbf{b}) = N$;
4: $\mathbf{B} = \{\mathbf{b}\}$;
5: **while** $\mathbf{B} \neq \emptyset$ **do**
6:     $\mathbf{B}^+ = \emptyset$;
7:     **for** each $\mathbf{b} \in \mathbf{B}$ **do**
8:         **if** $f(\mathbf{b}) < f^*$ **then**
9:             **if isSchedulable(b) then**
10:                 $\mathbf{b}^* = \mathbf{b}$;
11:                 $f^* = f(\mathbf{b})$;
12:             **else**
13:                 **for** $j = 1 : deg(\mathbf{b})$ **do**
14:                     $\mathbf{b}^+ = \mathbf{b}$;
15:                     set constellation size $\mathbf{b}^+(N - j + 1)$ to next value;
16:                     $deg(\mathbf{b}^+) = j$;
17:                     $\mathbf{B}^+ = \mathbf{B}^+ + \{\mathbf{b}^+\}$;
18:                 **end for**
19:             **end if**
20:         **end if**
21:     **end for**
22:     $\mathbf{B} = \mathbf{B}^+$;
23: **end while**

assignment of the degree to each constellation size vector assures that the algorithm generates a particular vector $\mathbf{b}$ only once, and all possible vectors in the case of no pruning, as proven in Lemma 3. Algorithm starts with the constellation size of minimum power consumption for each node, resulting in the vector $\mathbf{b} = (b_{11}, b_{21}, ..., b_{N1})$ (Line 2). Degree of root vector $(b_{11}, b_{21}, ..., b_{N1})$ is set to N (Line 3). Vector set $\mathbf{B}$ is defined as the set of constellation size vectors that are evaluated in the current iteration of the algorithm and initially contains the vector $(b_{11}, b_{21}, ..., b_{N1})$ (Line 4). f^* denotes the minimum value of objective function corresponding to a feasible constellation size vector and initialized to ∞ (Line 1). For each vector $\mathbf{b}$ in $\mathbf{B}$, the algorithm first determines whether it can improve the best solution so far with a smaller value of objective function (Lines $7-8$). If so, the algorithm checks the schedulability of vector $\mathbf{b}$. If $\mathbf{b}$ is also schedulable, the best constellation size vector $\mathbf{b}^*$ and best solution f^* are updated with $\mathbf{b}$ and the value of the objective function corresponding to $\mathbf{b}$, respectively (Lines $9-11$). Note that the vectors that do not improve the best solution and the schedulable vectors that improve the best solution are not branched into new vectors. If $\mathbf{b}$ is not schedulable, the algorithm branches the vector $\mathbf{b}$ into $deg(\mathbf{b})$ vectors (Lines $12-19$). For every j value in $[1, deg(\mathbf{b})]$ interval, a vector $\mathbf{b}^+$ is generated by setting the constellation size of the $N - j + 1$-th value in vector $\mathbf{b}$ to the next constellation size and the degree to j. Each newly generated vector $\mathbf{b}^+$ is included in the set $\mathbf{B}^+$, which will be equalized to the set $\mathbf{B}$ at the end for the evaluation in the next iteration of the algorithm (Lines 17 and 22). Algorithm terminates when all the vectors in $\mathbf{B}$ are schedulable or have an objective value greater than or equal to the best solution so far, generating $\mathbf{B} = \emptyset$ in the following iteration (Line 5). OFE algorithm is illustrated with an example in Fig. 3.

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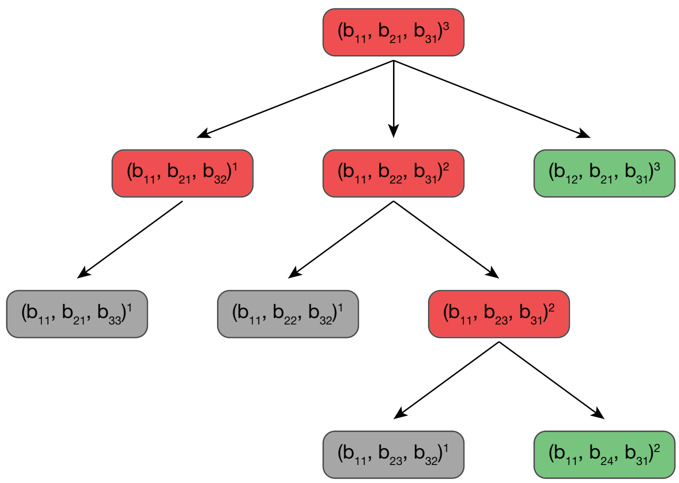

Fig. 3: OFE algorithm illustration for the case where $N = 3$ and $A_i = 5$, for all $i \in [1, N]$. The constellation size vectors are evaluated in the following order: (b_{11}, b_{21}, b_{31}), (b_{11}, b_{21}, b_{32}), (b_{11}, b_{22}, b_{31}), (b_{12}, b_{21}, b_{31}), (b_{11}, b_{21}, b_{33}), (b_{11}, b_{22}, b_{32}), (b_{11}, b_{23}, b_{31}), (b_{11}, b_{23}, b_{32}), (b_{11}, b_{24}, b_{31}). The superscripts represent the degree of the constellation size vectors. Green-colored constellation size vectors are not branched into new vectors since they are evaluated as schedulable. Grey-colored constellation size vectors are the vectors that are not branched into new vectors since their objective is greater than or equal to the best solution so far. Red-colored constellation size vectors are branched into new vectors since they are not schedulable and their objective is less than the best solution obtained so far.

***Lemma** 3:* In the OFE algorithm, each vector is generated only once and all possible constellation size vectors are generated in the case of no pruning.

Proof: OFE algorithm generates a descendant vector $\mathbf{b}^+$ of a particular vector $\mathbf{b}$ with $deg(\mathbf{b})$ by changing the constellation size $\mathbf{b}(N - j + 1)$ to the next value for a particular $j \in [1, deg(\mathbf{b})]$ (Lines $13 - 15$). The degree of a descendant node $\mathbf{b}^+$ generated by changing $\mathbf{b}(N - j + 1)$ is set to j (Line 16). Therefore, the degree of the nodes either decrease or stay constant on a particular path including all the subsequent descendant nodes starting with the root constellation size vector and ending in a particular constellation size vector. Hence, the degree of a particular vector $\mathbf{b}$ except the root constellation size vector is equal to minimum j such that $\mathbf{b}(N - j + 1)$ is not equal to $b_{(N-j+1)1}$ and the unique ascendant of that vector from which it is generated can be determined by setting the constellation size $\mathbf{b}(N - j + 1)$ to the lower value reversing the generation rule of descendant nodes.

Now, consider any possible constellation size vector $\overline{\mathbf{b}}$ except the root constellation size vector and let $j \in [1, N]$ be the degree of vector $\overline{\mathbf{b}}$. $\overline{\mathbf{b}}(i)$ is equal to $b_{(i)1}$ for all $i \in [N - j + 2, N]$ and $\overline{\mathbf{b}}(N - j + 1)$ is not equal to $b_{(N-j+1)1}$ from the previous argument. The ascendant of $\overline{\mathbf{b}}$ can be found by setting $\overline{\mathbf{b}}(N-j+1)$ to the lower constellation size value. If the lowered value is equal to $b_{(N-j+1)1}$, which will eventually happen on the path, the degree of the ascendant will be larger than j. Subsequent ascendants of the ascendant vector can be determined similarly. As long as any element $i \in [1, N - j + 1]$ of a particular vector $\mathbf{b}$ on the path is not equal to $b_{(i)1}$, the ascendant of $\mathbf{b}$ can be determined by setting $\mathbf{b}(N - deg(\mathbf{b}) + 1)$ to the lower constellation size value. This eventually leads to the root constellation size vector whose all elements $i \in [1, N]$ are equal to $b_{(i)1}$. Since each vector except the root constellation size vector has a unique ascendant, a unique path from any possible vector $\overline{\mathbf{b}}$ to the root constellation size vector exists. This completes the proof.□

Lemma 3 ensures that OFE algorithm will find the optimal constellation size vector in finite time if there exists a schedulable constellation size vector. The complexity of the OFE algorithm is $O(\prod_{i\in[1,N]} A_i \times F)$, where F is the complexity of the schedulability analysis for the specific scheduling algorithm used, since in the worst case the algorithm evaluates all possible constellation size vectors and checks the schedulability of each vector. For EDF scheduling algorithm, the complexity required by the exact schedulability analysis is given by $F = N \sum_{i=1}^{N} \frac{\min\{\frac{c}{1-c} \max_{i\in[1,N]}\{h_i - \Delta\}, \Omega\}}{h_i}$, where $c = \sum_{i=1}^{N} \frac{d_i(b_i)}{h_i}$ [23]. A categoric analysis of the schedulability of other scheduling algorithms can be found in [33].

V. Polynomial-time Heuristic Algorithms

In the previous section, we have proposed an optimal algorithm that efficiently spans the search space of the constellation size vectors till the optimality of a constellation size vector is guaranteed. Although the OFE algorithm reaches the optimality without searching whole search space using the relations among the constellation size vectors, the worst case complexity of the algorithm is still exponential. Therefore, for large network sizes and modulation schemes allowing higher number of bits per symbol, the OFE algorithm may require unreasonable runtimes. This situation is expectable as the wireless sensor network applications proliferate and communication systems provide higher data rates. Hence, in practical scenarios, the use of the OFE algorithm may be limited and faster solutions may be desired depending on the application at the expense of achieving sub-optimal results. This necessitates the design of efficient heuristic algorithms.

In the following, we first present a technique that reduces the problem search space significantly by first defining and exploiting utilization and energy consumption based dominance relations of the constellation size of each sensor node separately in Section V-A. We then propose two polynomial time heuristic algorithms based on moving either in the direction of maximum improvement in the power consumption related objective function while keeping feasibility starting with the constellation size vector corresponding to the maximum power consumption or in the direction of minimum degradation in the objective function searching for the feasibility starting with the constellation size vector associated with the minimum power consumption in Sections V-B and V-C, respectively.

A. Utilization based Search Space Reduction (USR)

***Definition** 1:* The *utilization* $u_i(b_i)$ of the constellation size value b_i of a sensor node i is defined as the ratio of its transmission delay $d_i(b_i)$ to its sampling period $h_i(b_i) = \frac{\Omega}{k_i^*(b_i)}$; i.e., $u_i(b_i) = \frac{d_i(b_i) k_i^*(b_i)}{\Omega}$.

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Utilization is a very important metric used commonly in determining the schedulability of real-time periodic tasks [35], [36]. In the real-time scheduling of periodic task sets, the utilization u_i of task i with execution time c_i and task period t_i is expressed as $u_i = \frac{c_i}{t_i}$. The utilization of a task in a real-time system is the fraction of time used by that task in the scheduling frame. Note that task i refers to the transmission of sensor node i, and c_i and t_i correspond to transmission time d_i and sampling period h_i, respectively, in our model. The total utilization U of a periodic task set is then defined as the sum of the utilizations of tasks; i.e., $U = \sum_i u_i$. This can again be interpreted as the fraction of time used by the entire periodic task set. Clearly, for $U > 1$, there exists no feasible schedule for the periodic task set with any scheduling algorithm. On the other hand, if $U \leq 1$, there may exist a feasible schedule depending on the parameters of the task set and the scheduling algorithm used in the system.

The total utilization of a periodic task set is used to determine the schedulability conditions for commonly used scheduling algorithms. For Rate Monotonic (RM) scheduling algorithm, which is the main fixed priority algorithm used in the real time scheduling of periodic task sets, where the tasks are assigned fixed priorities such that the task with smaller period receives higher priority, a sufficient schedulability condition for a set of n tasks is defined as

$$\sum_{i=1}^{n} u_i \leq n(2^{1/n} - 1) \tag{11}$$

under the assumption that the relative deadlines of the tasks are equal to their periods. For EDF scheduling algorithm, which is the main dynamic priority algorithm in the real time scheduling of periodic task sets, where priorities are assigned dynamically and are inversely proportional to the absolute deadlines of the tasks, a set of n tasks is schedulable if and only if

$$\sum_{i=1}^{n} u_i \leq 1 \tag{12}$$

under the assumption that the relative deadlines of the tasks are equal to their periods. Similar schedulability conditions employing only the utilizations of the tasks for periodic task systems can be found in [35], which demonstrates the significance of the utilization in the schedulability analysis of the real time periodic systems.

While utilization based schedulability conditions are mainly based on the assumption that the deadlines of the tasks are equal to their periods, the exact schedulability analysis for periodic task systems without this assumption employ different task parameters in addition to their utilization. For instance, Response Time Analysis (RTA) proposed for the RM algorithm and Processor Demand Criterion (PDC) technique proposed for the EDF algorithm describe exact schedulability conditions employing execution time, deadline and periods of the tasks [35], [36]. However, the following results on the sustainability of these schedulability analyses present a strong basis to employ utilization as a heuristic to evaluate the impact of a single task on the schedulability of a periodic task set, or for our system, the impact of the utilization of a sensor node on the schedulability of the entire network of nodes. A schedulability test for a scheduling algorithm is defined to be sustainable if any system deemed schedulable by that schedulability test remains schedulable when the parameters of one or more individual tasks are changed in a certain direction. Both RTA proposed for RM schedulability and PDC proposed for EDF schedulability are sustainable with respect to execution requirements, task periods and deadlines as presented in Theorems 1 and 10 of [36], respectively. The sustainability of these exact schedulability tests indicates that decreasing the transmission delay or increasing the sampling period of a sensor node keeps the schedulability of a system if the original system is schedulable. Considering the definition of utilization, given by Definition 1, which is the ratio of transmission delay to sampling period, both decreasing transmission delay and increasing sampling period map to a decreasing utilization for a sensor node. Hence, although not necessarily, a decreasing utilization for a sensor node is expected to preserve the schedulability of a system along with the fact that a smaller utilization corresponds to a smaller fraction of time allocated by that sensor node in the entire schedule. Considering this and the utilization based schedulability conditions discussed previously leads us to use the utilization to define a dominance relation among the constellation sizes of a sensor node and propose a search space reduction algorithm that will reduce the number of constellation sizes to be considered for a sensor node.

***Definition** 2:* A constellation size value $b_i^{'}$ for a sensor node i is said to be *dominating* another constellation size value $b_i^{''}$ if

1) the power consumption corresponding to $b_i^{'}$ is less than or equal to the power consumption corresponding to $b_i^{''}$; i.e., $W_i(b_i^{'}, k_i^*(b_i^{'})) \leq W_i(b_i^{''}, k_i^*(b_i^{''}))$,
2) the utilization corresponding to $b_i^{'}$ is less than or equal to the utilization corresponding to $b_i^{''}$; i.e., $u_i(b_i^{'}) \leq u_i(b_i^{''})$.

Algorithm 2 Utilization based Search Space Reduction (USR) Algorithm

Input: $b_i^{\min}$,$b_i^{\max}$, $A_i = b_i^{\max} - b_i^{\min} + 1,\ i \in [1, N]$
Output: $b_{ij},\ i \in [1, N],\ j \in [1, C_i]$

for $i = 1 : N$ **do**
  order $b_i \in [b_i^{\min}, b_i^{\max}]$ in increasing $W_i(b_i, k_i^*(b_i))$: $S_i^p = [b_{i1}^p, ..., b_{iA_i}^p]$;
  order $b_i \in [b_i^{\min}, b_i^{\max}]$ in increasing $u_i(b_i)$: $S_i^u = [b_{i1}^u, ..., b_{iA_i}^u]$;
  $C_i = 1$;
  while $S_i^p \neq \emptyset$ **do**
    $b_{iC_i} = S_i^u(C_i)$;
    determine k s.t. $b_{ik}^p = b_{iC_i}$;
    remove $\{b_{ij}^p | j \geq k\}$ from S_i^p;
    remove $\{b_{ij}^p | j > k\}$ from S_i^u;
    $C_i = C_i + 1$;
  end while
end for

Utilization based Search Space Reduction (USR) Algorithm is based on reducing the number of constellation size values corresponding to each sensor node separately by exploiting

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the dominance relations among the constellation size values, as described in detail next (Algorithm 2). For each sensor node $i \in [1, N]$, the algorithm starts by sorting the corresponding constellation size values in the feasible interval $[b_i^{\min}, b_i^{\max}]$ in increasing power consumption $W_i(b_i, k_i^*(b_i))$ and utilization $u_i(b_i)$, and storing them in the sets S_i^p and S_i^u, respectively (Lines $2-3$). Here, b_{ij}^p and b_{ij}^u denote the constellation size with the j-th smallest power consumption and utilization, respectively. The algorithm initializes C_i to 1 and increases C_i by 1 at each iteration of the algorithm such that the constellation size corresponding to the C_i-th smallest utilization in S_i^u is considered at the C_i-th iteration (Line $4, 6, 10$). At the C_i-th iteration, the index of b_{iC_i} in the set S_i^p is determined and denoted by k (Line 7). Based on Definition 2, b_{iC_i} optimally dominates all the constellation sizes $\{b_{ij}^p | j > k\}$ since it has both lower utilization and energy consumption. $\{b_{ij}^p | j > k\}$ therefore need not be considered in the rest of the algorithm and are removed from S_i^u (Line 9). The total number of iterations corresponding to each node i given by the value of C_i at the output of the algorithm then gives the reduced number of constellation sizes for consideration. The algorithm stops when all the constellation sizes are either included in the output of the algorithm or removed by considering dominance relations (Lines $5, 8$).

TABLE I: Example search space reduction by USR.

No	S_i^u	S_i^p
0	$\{4, 10, 9, 3, 7, 8, 6, 1, 2, 5\}$	$\{3, 5, 9, 8, 2, 4, 1, 10, 6, 7\}$
1	$\{4, 9, 3, 8, 2, 5\}$	$\{3, 5, 9, 8, 2\}$
2	$\{4, 9, 3, 5\}$	$\{3, 5\}$
4	$\{4, 9, 3\}$	$\emptyset$

Consider the example execution of the algorithm in Table I. Let $b_i^{\min} = 1$ and $b_i^{\max} = 10$ for sensor node i and the sorted lists of the constellation sizes in increasing orders of utilization and power consumption be given by $\{4, 10, 9, 3, 7, 8, 6, 1, 2, 5\}$ and $\{3, 5, 9, 8, 2, 4, 1, 10, 6, 7\}$, respectively. At the first iteration of the algorithm, $b_{iC_i} = 4$ gives minimum utilization. The constellation size values $\{1, 10, 6, 7\}$ yield both greater utilization and power consumption than 4. Hence, these values are removed from S_i^u, resulting in $\{4, 9, 3, 8, 2, 5\}$, and together with $b_{iC_i} = 4$ value itself from S_i^p, resulting in $\{3, 5, 9, 8, 2\}$. At the second iteration of the algorithm, the constellation size value with the second minimum utilization, given by $b_{iC_i} = 9$, is considered. The constellation size values $\{8, 2\}$ yield both greater utilization and power consumption. S_i^u is updated to exclude these values, resulting in $\{4, 9, 3, 5\}$, and S_i^p is updated by excluding these values and the constellation size 9, resulting in $\{3, 5\}$. At the last iteration of the algorithm, when the constellation size value with the third minimum utilization, given by $b_{iC_i} = 3$, is considered, both the optimally dominating set given by $\{5\}$ and the constellation size itself equal to 3 are excluded from S_i^p. S_i^p then becomes empty set, which is the end of the algorithm for sensor node i. When we order the constellation size values in the resulting set $S_i^u = \{4, 9, 3\}$ in increasing power consumption, the resulting set is given by $\{3, 9, 4\}$. These sets are reversed versions of each other. This result is given as a Lemma next and exploited in the heuristic algorithms in Sections V-B and V-C.

Lemma 4: In the set S_i^u at the output of the USR algorithm, in the direction of increasing power consumption, the utilization decreases.

Proof: S_i^u initially consists of all possible constellation size values $j \in [1, A_i]$ in increasing order of utilization. At each iteration of the algorithm, the algorithm eliminates the constellation size values from the set S_i^u with greater indices and higher power consumptions than the constellation size considered at that iteration. Hence, for any constellation size in the set S_i^u at the output of USR algorithm, the constellation sizes with greater indices have lower power consumptions. Therefore, in the direction of increasing utilization, the power consumption decreases and vice versa. □

B. Keep Feasible Improve Maximum (KFIM) Algorithm

Algorithm 3 **Keep Feasible Improve Maximum (KFIM) Algorithm**

Input: $b_{ij},\ \forall i \in [1, N],\ \forall j \in [1, C_i]$;

```
1:  b = (b_11, b_21, ..., b_N1);
2:  if isSchedulable(b) then
3:      while b ≠ (b_1C_1, b_2C_2, ..., b_NC_N) do
4:          j = argmin_{i | b_i^{++} ≠ b} f(b_i^{++});
5:          if isSchedulable(b_j^{++}) then
6:              b = b_j^{++};
7:          else
8:              break;
9:          end if
10:     end while
11:     return b;
12: else
13:     return no feasible solution exists;
14: end if
```

Keep Feasible Improve Maximum (KFIM) algorithm, given by Algorithm 3, is described as follows. Let the i-th element of the constellation size vector $\mathbf{b}$ given by $\mathbf{b}_i$ correspond to the constellation size of node i. Let $\mathbf{b}_i^{++}$ denote the constellation size vector obtained by keeping all the elements of the vector $\mathbf{b}$ the same except the i-th element, which is set to the next constellation size in the increasing utilization direction if it is not equal to b_{iC_i} and also kept the same otherwise. The algorithm starts with the constellation size vector $\mathbf{b}$ corresponding to the minimum utilization and maximum power consumption for each node in the network; i.e., $\mathbf{b}_i = b_{i1}$ for all $i \in [1, N]$ (Line 1). If this vector is not schedulable, then the algorithm terminates stating no feasible solution exists since the utilization of each sensor node is at minimum (Lines $12-13$). If it is schedulable, then the algorithm determines the sensor node j that minimizes the objective value f when the constellation size of that sensor node is set to the next value in the direction of increasing utilization or, in other words, decreasing power consumption (Line $2, 4$). If the resulting constellation size vector $\mathbf{b}_j^{++}$ is schedulable, then it is set to the current best solution $\mathbf{b}$ (Lines $5-6$). The algorithm terminates if $\mathbf{b}_j^{++}$ is not schedulable or $\mathbf{b} = (b_{1C_1}, b_{2C_2}, ..., b_{NC_N})$ (Lines 3 and $7-8$).

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C. Seek Feasible Degrade Minimum (SFDM) Algorithm

Algorithm 4 **Seek Feasible Degrade Minimum (SFDM) Algorithm**

Input: $b_{ij},\ \forall i \in [1, N],\ \forall j \in [1, C_i]$;

```
1: b = (b_{1C_1}, b_{2C_2}, ..., b_{NC_N});
2: while b ≠ (b_{11}, b_{21}, ..., b_{N1}) do
3:    if isSchedulable(b) then
4:       break;
5:    else
6:       j = argmin_{ {i | b_i^{--} ≠ b} } f(b_i^{--});
7:       b = b_j^{--};
8:    end if
9: end while
10: if isSchedulable(b) then
11:    return b;
12: else
13:    return no feasible solution exists;
14: end if
```

Seek Feasible Degrade Minimum (SFDM) algorithm, given by Algorithm 4, is described as follows. Let $\mathbf{b}_i^{--}$ denote the constellation size vector obtained by keeping all the elements of the vector $\mathbf{b}$ the same except the i-th element, which is set to the next constellation size in the decreasing utilization direction if it is not equal to b_{i1} and also kept the same otherwise. The algorithm starts with the constellation size vector $\mathbf{b}$ corresponding to the minimum power consumption and maximum utilization for each node in the network; i.e., $\mathbf{b}_i = b_{iC_i}$ for all $i \in [1, N]$. If this constellation size vector is schedulable, the algorithm terminates and solution is returned (Lines $3-4$ and $10-11$). Otherwise, the algorithm seeks a feasible solution in the direction of minimum increase in the value of the objective function (Lines $5-7$). At each iteration, the algorithm determines the sensor node j that minimizes the objective value f when the constellation size of that sensor node is set to the next value in the direction of increasing power consumption or, in other words, decreasing utilization. The resulting constellation size vector $\mathbf{b}_j^{--}$ is set to the current solution $\mathbf{b}$. The algorithm terminates if the resulting $\mathbf{b}$ is schedulable or $\mathbf{b} = (b_{11}, b_{21}, ..., b_{N1})$ (Lines $2-4$).

The complexity of both heuristic algorithms described in Sections V-B and V-C is $O(\sum_{i\in[1,N]} C_i \times F)$ since they check the schedulability of the constellation size vector with complexity $O(F)$ and vary the constellation size of one node at each iteration, resulting in $\sum_{i\in[1,N]} C_i$ iterations at maximum.

VI. Performance Evaluation

The goal of this section is to evaluate the performance of the proposed heuristic algorithms compared to the traditional separate design of controller and communication systems, previously proposed heuristic algorithms and optimal solution for different network sizes, modulation schemes, objective functions, and control system parameters. In the separate design of controller and communication systems, denoted by "TS", the constellation size and sampling period of the sensor nodes are predetermined such that a feasible solution is guaranteed for the worst case scenario. For example, WirelessHart [8] and ISA100.11a [7], which are the two competing wireless standards for industrial control applications, employ O-QPSK (Offset Quadrature Phase Shift Keying) without including any mechanism for the optimization of constellation size and sampling period. The heuristic algorithm previously proposed for the joint optimization of controller and communication systems with the objective of minimizing the total power consumption of the wireless network while guaranteeing the performance and stability of the control system and the schedulability of the communication system for MQAM modulation and EDF scheduling, is denoted by "HS" [23]. This algorithm is based on reducing the optimization problem to an IP problem based on the analysis of the optimality conditions, solving the Linear Programming (LP) relaxation of the IP problem formulation and ceiling each element of the resulting solution vector to obtain an integral solution while avoiding the violation of the schedulability constraint. The optimal solution is obtained by the OFE algorithm and denoted by "OFE". The proposed heuristic algorithms that combine utilization based search space reduction algorithm, denoted by USR, with efficient search algorithms, denoted by KFIM and SFDM, are called "USR-KFIM" and "USR-SFDM", respectively. For schedule construction and schedulability analysis, we use EDF scheduling algorithm. Since we have obtained similar behavior for the scheduling algorithms designed for maximum adaptivity in [14], we have not included the corresponding simulation results.

Simulation results are obtained by averaging the performance over 1000 independent random network topologies where the sensor nodes are uniformly distributed within a circular area and transmit to a controller located in the center of the area. The channel attenuation is calculated by using Rayleigh fading with scale parameter set to the mean power level determined by using the large scale statistics modeled as $PL(d) = PL(d_0) + 10\alpha\log(d/d_0) + Z$, where d is the distance between the node and the controller, $PL(d)$ is the path loss at distance d, $PL(d_0)$ is the path loss at reference distance d_0, α is the path loss exponent, and Z is Gaussian random variable with zero mean and standard deviation σ_z [37], [38]. Each sensor node i has a packet length of L_i bits to be transmitted periodically. Table-II lists the parameters used in the simulations.

$W^{t,\max}$	250 mW	W^c	50mW
$L_i, i \in [1, N]$	100 bits	δ	0.95
$PL(d_0)$	70 dB	d_0	1 m
α	3.5 [27]	σ_z	4 dB [37]

TABLE II: Simulation Parameters

Fig. 4 shows the total power consumption of the algorithms for different number of nodes, where the modulation scheme is MQAM, the objective function is the total power consumption as given in Eq. (6a), the nodes are uniformly distributed within a circular area of radius 5 m, MAD and MATI values are chosen as $\Delta = 25$ ms and $\Omega = 25$ ms. The power consumption of the TS algorithm increases linearly with the number of nodes, as expected, since the constellation size is fixed at a predetermined value. On the other hand, the power

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consumption of the remaining algorithms increases linearly only up to a specific value, which is around 25 nodes in this case, since the objective of minimizing the power consumption of each node in the network separately without considering the schedulability constraint achieves the objective of minimum total power consumption. However, as the number of nodes increases further, the nodes need to make a joint decision to satisfy the schedulability constraint. The schedulability constraint forces the nodes to choose a smaller constellation size than they would select independently of the other nodes. This accelerates the increase in the power consumption. The acceleration effect of the HS algorithm is greater than that of the proposed USR-KFIM and USR-SFDM algorithms, moving the resulting power consumption value away from the optimal solution dramatically. The main reason for the lower performance of the HS algorithm is that the possibly non-integer optimal constellation size values resulting from the LP relaxation of the IP problem formulation are ceiled to obtain an integral solution while avoiding the violation of the schedulability constraint. The proposed USR-KFIM and USR-SFDM algorithms, however, intelligently search for the minimum power consumption still performing very close to the optimal solution as the number of nodes increases. The average approximation ratio values of the USR-KFIM and USR-SFDM algorithms are below 1.01, where the approximation ratio is defined as the ratio of the value of the objective function of a particular algorithm to the optimal solution.

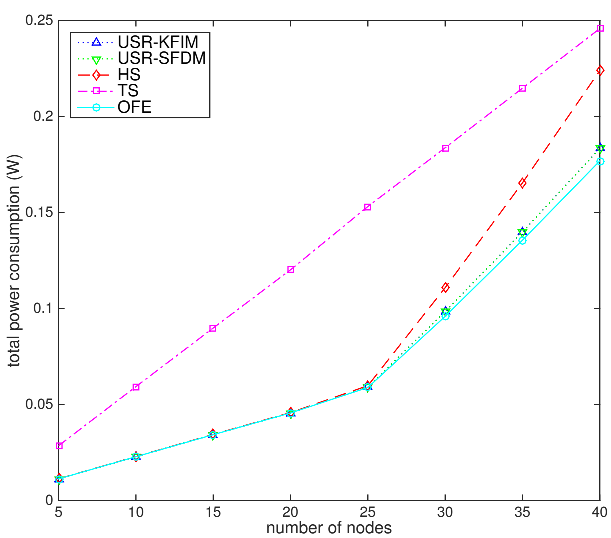

Fig. 4: Total power consumption for different number of nodes where the modulation scheme is MQAM, $\Delta = 25$ ms and $\Omega = 25$ ms.

Fig. 5 shows the average runtime of the algorithms for different number of nodes in the same scenario as Fig. 4. The proposed USR-KFIM and USR-SFDM algorithms provide better performance at lower average runtime than the previously proposed HS algorithm due to mainly considerable decrease in the search space for the constellation size vectors by the usage of the proposed USR technique. The average runtime of USR-KFIM and USR-SFDM increases almost linearly as the number of nodes increases. On the other hand, the average runtime of the OFE algorithm increases exponentially as the number of nodes increases and is much more than the average runtime of the heuristic algorithms.

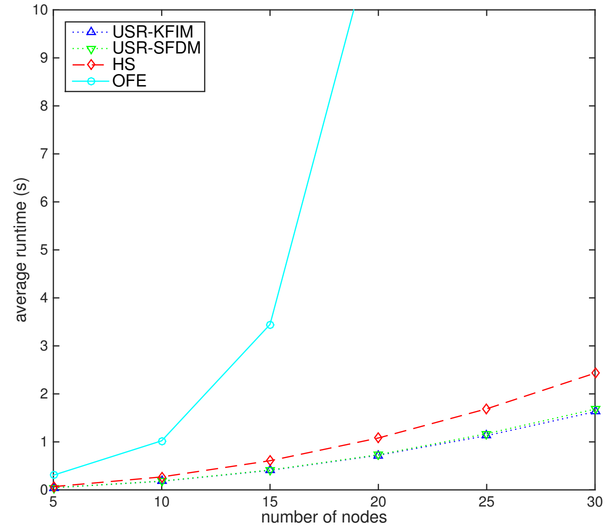

Fig. 5: Average runtime for different number of nodes where the modulation scheme is MQAM, $\Delta = 25$ ms and $\Omega = 25$ ms.

Figs. 6, 7 and 8 show the performance of the proposed USR-KFIM and USR-SFDM algorithms compared to the TS and optimal OFE algorithms for different scenarios where the previously proposed HS algorithm cannot be used to find a solution. The objective function in Figs. 6 and 8 is the log-sum of the node power consumptions given in Eq. (6b) whereas that in Fig. 7 is total power consumption. The modulation in Figs. 7 and 8 is MFSK, whereas that in Fig. 6 is MQAM. The nodes are uniformly distributed within a circular area of radius 5 m. MAD and MATI values are chosen as $\Delta = 25$ ms and $\Omega = 25$ ms. The performance of the algorithms is very similar to that depicted in Fig. 4: The proposed heuristic algorithms perform much better than the TS algorithm and very close to optimal yielding average approximation ratio values below 1.01. This illustrates the robustness of the proposed algorithms to different modulation schemes and objective functions.

Fig. 9 shows the total power consumption of the algorithms in a network of 20 nodes for different MAD values where the modulation scheme is MQAM, the objective is minimizing total power consumption, the nodes are uniformly distributed within a circular area of radius 10 m and the MATI value is chosen as $\Omega = 200$ ms. The power consumption of the TS algorithm stays constant independent of the MAD requirement since it employs a fixed constellation size vector determined to guarantee feasibility for all MAD values. The power consumption of the remaining algorithms is significantly lower than that of the TS algorithm. As the MAD increases up to a certain value, around 2 ms in this case, the total power consumption decreases. Beyond this certain MAD value, however, the total power consumption stays constant since the optimal constellation size does not change although the feasible region expands. The average approximation ratio of the proposed USR-KFIM and USR-SFDM algorithms is less than 1.01 and outperforms

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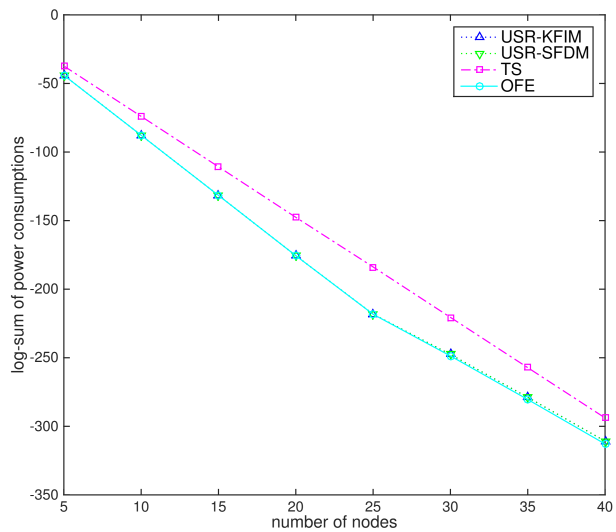

Fig. 6: Log-sum of the node power consumptions for different number of nodes where the modulation scheme is MQAM, $\Delta = 25$ ms and $\Omega = 25$ ms.

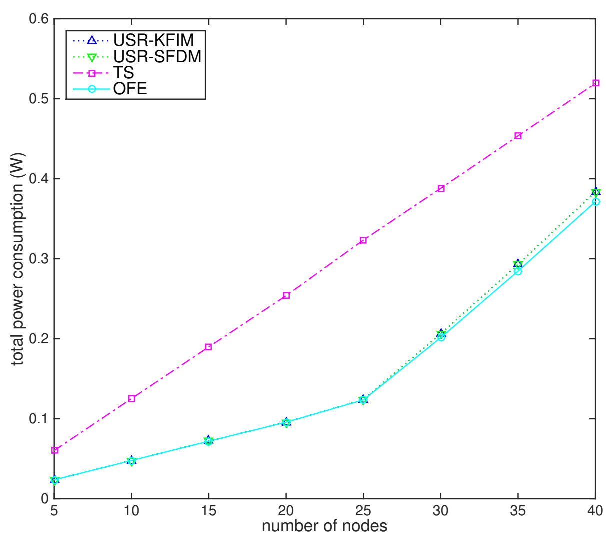

Fig. 7: Total power consumption for different number of nodes where the modulation scheme is MFSK, $\Delta = 25$ ms and $\Omega = 25$ ms.

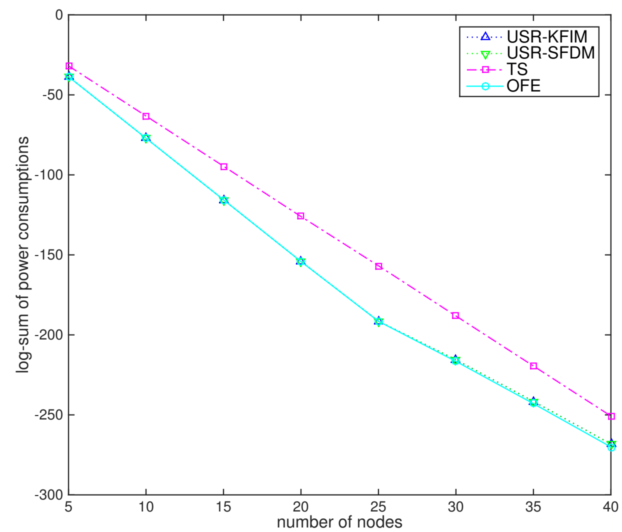

Fig. 8: Log-sum of the node power consumptions for different number of nodes where the modulation scheme is MFSK, $\Delta = 25$ ms and $\Omega = 25$ ms.

the HS algorithm for all MAD values. The increasing gap between the USR-KFIM and USR-SFDM algorithms, and HS algorithm as MAD decreases mainly results from the shrinkage in the feasible region. The smaller feasible region contains the constellation size vectors with higher power consumption values. Then the constellation size vectors determined by the inefficient mechanism of the HS algorithm based on the LP relaxation and ceiling results in a larger difference from the optimal value compared to the USR-KFIM and USR-SFDM algorithms with efficient search mechanisms.

Fig. 10 shows the total power consumption of the algorithms in a network of 20 nodes for different MATI values where the modulation scheme is MQAM, the objective is minimizing total power consumption, the nodes are uniformly distributed within a circular area of radius 10 m and the MAD value is chosen as $\Delta = 10$ ms. The effect of the MATI on the total power consumption is twofold. The power consumption is inversely proportional to MATI, as given in Eq. (3). The TS algorithm employing a fixed constellation size vector guaranteeing the feasibility for all MATI values illustrates only this functional dependency on the MATI. However, the other algorithms depicts the additional effect of the schedulability constraint on the total power consumption. As MATI decreases, the feasible region of constellation size vectors shrinks due to the schedulability constraint, requiring a joint decision among the nodes, which increases power consumption even further. Moreover, the performance of the proposed USR-KFIM and USR-SFDM algorithms is robust to varying MATI values and very close to optimal with an average approximation ratio of around 1.01, outperforming the HS algorithm yielding an approximation ratio of around 1.05. The gap between the USR-KFIM and USR-SFDM algorithms, and HS algorithm increases as MATI decreases. This is mainly due to the LP relaxation and following ceiling operation used in the HS algorithm, which is inefficient in finding the optimal solution in scenarios requiring joint decision among the nodes.

VII. Conclusion

In this paper, we study a joint optimization framework for the design of communication and control systems in WNCS considering the high reliability and strict delay constraints of control systems, the limited battery resources of sensor nodes, the non-zero packet error probability and delay of wireless transmissions. We have generalized our previous work on the joint optimization of communication and control systems in WNCS with the objective of minimizing the total power consumption of the network for MQAM modulation and EDF scheduling. The generalization comprises a wide range of objective functions including total power consumption of the

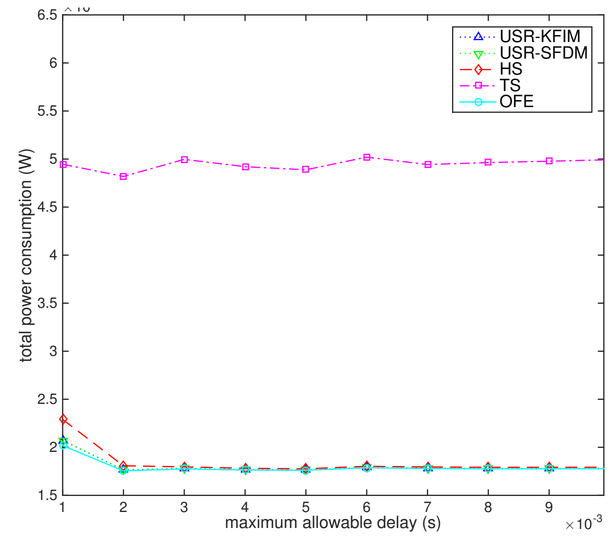

Fig. 9: Total power consumption in a network of 20 nodes employing MQAM modulation and $\Omega = 200$ ms for different MAD values.

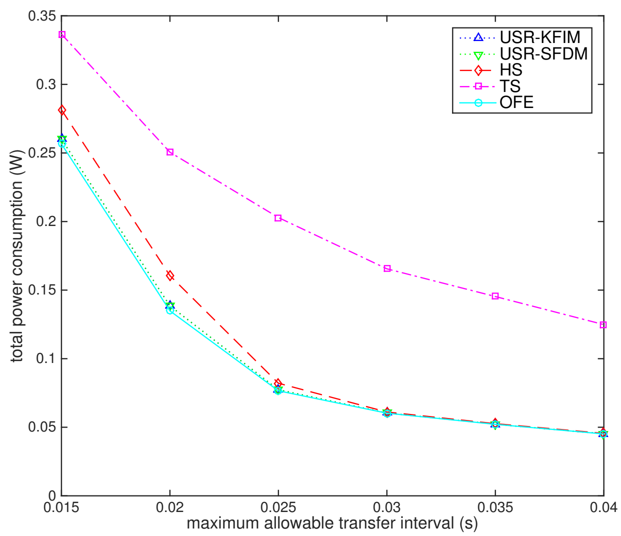

Fig. 10: Total power consumption in a network of 20 nodes employing MQAM modulation and $\Delta = 10$ ms for different MATI values.

network, maximum power consumption among the nodes in the network and log-sum of the power consumptions of the nodes in the network, any modulation scheme and any scheduling algorithm. We first propose an exact but efficient smart enumeration based algorithm for the generalized problem by exploiting the optimality conditions on the decision variables. We then propose two polynomial time heuristic algorithms based on a search space reduction technique that exploits the utilization and energy consumption dominance relations of the constellation size of each sensor node and smart searching technique that proceeds by evaluating the feasibility conditions and objective function of neighboring constellation size vectors. Extensive simulations illustrate that the proposed heuristic algorithms perform very close to optimal and much better than the existing methods at smaller runtime for various network sizes, modulation schemes, objective functions, and control system parameters. In the future, we plan to extend this framework for multi-hop and cellular networks.

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