Joint Optimization of Communication and Controller Components of Wireless Networked Control Systems

Yalcin Sadi, Sinem Coleri Ergen

Abstract—**Designing communication system for wireless networked control systems requires overcoming the additional challenge of maintaining a guaranteed performance for the control system in the presence of wireless network induced imperfections including packet error, delay, sampling and quantization errors compared to traditional wireless sensor networks. The joint optimization of controller and communication systems encompassing efficient abstractions of each system and taking into account all wireless induced imperfections, the parameters of the wireless communication system including the transmission power, rate and scheduling and the parameters of the control system including the sampling period has been studied for the objective of minimizing the average power consumption of the network and the MQAM modulation scheme. In this paper, we extend the joint optimization problem for a generalized power cost function that represents many power-related objectives including minimization of total power consumption of the network and minimization of maximum power consumption among the nodes in the network and for any modulation scheme that satisfies certain properties including MQAM and MFSK. The optimization problem is formulated as a Mixed-Integer Programming problem thus difficult to solve for the global optimum. However, upon determining the optimality conditions for the optimization variables, the problem reduces to an Integer Programming problem for which we propose an optimal fast enumeration algorithm. Simulations demonstrate that the proposed optimal solution method outperforms the traditional separate design of control and communication systems.**

Index Terms—**wireless communication, networked control system, optimization, energy minimization, stability**

I. Introduction

Wireless Networked Control Systems (WNCSs) are spatially distributed control systems in which sensors, actuators and controllers communicate through a wireless network [1]. Bringing many advantages such as the ease of installation and maintenance, low complexity and cost, and large flexibility to accommodate the modification and upgrade of the components, deployment of wireless communication in information transfer creates a tremendous potential in WNCSs to enhance the performance of many large-scale distributed systems including industrial automation [2], automated highway [3] and smart grid [4]. The application of wireless communication in the control applications and the studies on WNCSs have been actively supported by the leading industrial organizations [5], [6], [7].

Designing a WNCS is a very challenging task since control systems often have strict timing and reliability requirements which are generally hard to satisfy by wireless sensor networks due to the wireless communication induced imperfections such as non-zero packet error probability, non-zero delay and sampling and quantization errors. On the other hand, improving the performance of the control system requires decreasing the packet error probability, delay and sampling period which increases the energy consumption in the communication. This trade-off between the communication and controller system performances reveals the need for techniques for quantification of the joint performance of these systems in terms of the wireless communication parameters including the transmission power, rate and scheduling of the network nodes and the control parameters including the sampling period.

The studies on the communication system design for Networked Control Systems (NCS) have remained very limited due to the lack of efficient abstractions of the control and communication systems in a joint manner. This led to either simplistic problem formulations by exclusion of some of the main control and communication system parameters or numerical solutions for specific scenarios avoiding the widespread use of the techniques. While some studies [8], [9] focus on the scheduling optimization considering sampling period and delay requirements of the sensor nodes, some studies [10] focus on the sampling period and delay optimization with the objective of minimizing the overall performance loss. The solutions proposed in these works however are not applicable to WNCS since they assume zero packet error probability. Communication system design for WNCS is studied in [5], [6], [11] to achieve low end-to-end delay and control jitter in very large mesh networks. In [12], the energy consumption of the network is minimized considering the packet loss probability and delay distribution. The study in [13] maximizes the control system performance subject to the wireless link capacity constraints and delay requirement of the control system. However, none of these studies consider optimization of the key parameters of the wireless communication system including the transmission power and rate of the nodes.

The joint optimization of controller and communication systems taking into account all the wireless induced imperfections and the parameters of both the wireless communication system and the control system has been studied in [14]

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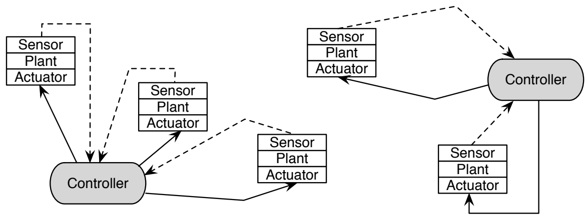

Fig. 1: Overview of the WNCS architecture.

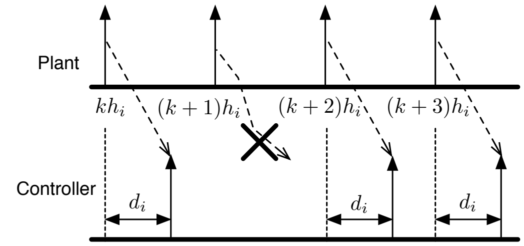

Fig. 2: Timing diagram between a plant and a controller communicating over a wireless network.

with the objective of minimizing the power consumption of the communication system guaranteeing the performance and stability of the control system and the schedulability in the communication system for MQAM modulation scheme. In this paper, our goal is to extend this study by generalizing the optimization problem for a wide range of objectives and modulation schemes satisfying certain properties and propose a general solution method that can be applicable for the studied generalized optimization problem. The original contributions of the paper are listed as follows:

- We provide a generalized optimization framework with a generalized power cost function as the objective for the joint optimization of controller and communication systems encompassing efficient abstractions of both systems.
- We propose an optimal solution method for the generalized optimization problem that can be applicable to a wide range of control system applications since we do not consider any specific objective or modulation scheme in the optimization problem.

The rest of the paper is organized as follows. Section II describes the control and communication system models and the assumptions used throughout the paper. The generalized joint optimization of controller and communication systems has been formulated and an optimal solution method has been proposed in Section III. Simulations are presented in Section IV. Finally concluding remarks are given in Section V.

II. System Model and Assumptions

The system model and assumptions are detailed as follows.

1) The system architecture for a WNCS is illustrated in Fig. 1. Multiple plants in the network where each plant has an attached sensor node are controlled through wireless communication. Each sensor samples the output of its associated plant periodically and forwards the samples to the corresponding controller of that particular plant. Since this is performed through wireless communication, wireless induced imperfections such as delays and packet errors naturally arise. Upon reception of the sampled measurements, the controller computes a control command to be forwarded to the actuator attached to the plant. One controller is assigned as the coordinator.
2) Information transfer beween a plant and its associated controller is depicted in Fig. 2. The sampling period of a node i, the transmission delay of the packet containing the samples of node i and the packet error probability are denoted by h_i, d_i, and p_i respectively. In order to maintain the arrival order of the packets to the controller correct, we assume that transmission delay of the packet must not exceed the sampling period of the node; i.e. $d_i \leq h_i$.
3) The packet error model is assumed to be a Bernoulli random process with probability p_i for node i for simplification.
4) Time Division Multiple Access (TDMA) is considered as MAC protocol since it provides both delay guarantee and energy efficiency for the networks with predetermined topology and data generation patterns [15]. TDMA is commonly used in industrial control applications [5], [6] having these characteristics.
5) The time is divided into scheduling frames of fixed lengths each of which is further partitioned into a beacon and variable number of variable-length time slots. Coordinator controller uses the beacon to maintain the synchronization among the elements of the network and to inform the network about the scheduling decisions including the transmission power, rate and sampling period of each sensor node.
6) We consider only the power consumption in the transmission of the packets since it is much larger than those in the sleep and transient modes [16].
7) The performance and stability conditions for the control systems have been formulated in the form of Maximum Allowable Transfer Interval (MATI) and Maximum Allowable Delay (MAD) constraints. MATI is defined as the maximum allowed time interval between subsequent state vector reports from the sensor nodes to the controller. MAD is defined as the maximum allowed packet delay for the transmission from the sensor node to the controller, in [17], [18]. Although it is possible to meet such strict real-time constraints in wireline networks, it is naturally infeasible in wireless networks due to non-zero packet error probability induced by wireless communication. To overcome this infeasibility problem, in many control applications such as wireless industrial automation [5], air transportation systems [19] and autonomous vehicular systems [20], MATI constraint has been replaced by stochastic MATI constraint which is keeping the time interval between subsequent state vector reports below the MATI value with a predefined

probability smaller than 1 to maintain the stability of control systems. Stochastic MATI constraint is formulated as

$$\Pr\left[\mu_i(h_i, d_i, p_i) \leq \Omega\right] \geq \delta \quad (1)$$

where μ_i is the time interval between subsequent state vector reports of node i as a function of h_i, p_i and d_i; Ω is the MATI; and δ is the minimum probability with which MATI should be achieved. The values of Ω and δ are determined by the control system. The number of reception opportunities of the state vector reports is equal to $\left\lfloor \frac{\Omega}{h_i} \right\rfloor$ within each Ω. Based on the assumption above on the modeling of packet error as a Bernoulli random process with probability p_i, Eq. (1) can be rewritten as

$$1 - p_i^{\left\lfloor \frac{\Omega}{h_i} \right\rfloor} \geq \delta \quad (2)$$

In [14], stochastic MATI constraint is used in the optimization framework to represent the stability requirement of the control systems.

8) In addition to the stochastic MATI constraint, MAD constraint is essential to maintain the performance and the stability of the control systems on a certain level [21]. It is expressed as

$$d_i \leq \Delta \quad (3)$$

where Δ is the MAD value to stabilize the control system. Typical Δ values are on the order of of a few tens of milliseconds for fast control applications [5], [22].

9) The average power consumption of a sensor node i is formulated as a function of the sampling period, transmission delay and packet error probability as

$$W_i\left(h_i, d_i(b_i), p_i\right) = \frac{\left(W_i^t\left(b_i, p_i\right) + W_i^c\right) d_i(b_i)}{h_i} \quad (4)$$

where b_i is the number of bits used per symbol or the constellation size given a predetermined modulation scheme and d_i is represented as a function of b_i for that particular modulation scheme, W_i^t is the transmission power calculated as a function of the parameters b_i and p_i for a given modulation and channel coding, and W_i^c is the circuit power consumption in the active mode at the transmitter. In the following, we will use the notation of $W_i\left(h_i, b_i, p_i\right)$ instead of $W_i\left(h_i, d_i(b_i), p_i\right)$ for convenience since the optimization variables will be the sampling period h_i, the constellation size b_i, and the packet error probability p_i.

10) We assume that, due to its limited weight and size, transmit power of a sensor node cannot exceed a maximum power level $W^{t,\max}$. The maximum transmit power constraint is formulated as

$$W_i^t(b_i, p_i) \leq W^{t,\max} \quad (5)$$

11) Schedulability constraint represents the allocation of the transmission times of the sensor nodes in the network. It is required to guarantee a feasible schedule for the determined set of constellation size and sampling period values for each node in the network given a MAD value and expressed as

$$\{d_i(b_i), h_i, \Delta\} \in S^{feasible} \quad (6)$$

where $S^{feasible}$ denotes the set of $\{d_i(b_i), h_i, \Delta\}$ values such that a feasible schedule can be constructed. Depending on the specific scheduling algorithm chosen, $S^{feasible}$ may change meaning that a set $\{d_i(b_i), h_i, \Delta\}$ can yied a feasible schedule using a particular scheduling algorithm while not yielding a feasible schedule using another scheduling algorithm. In [14], a schedulability constraint is proposed for pre-emptive Earliest Deadline First (EDF) scheduling algorithm to be used in the optimization problem. In this paper, we do not consider any particular scheduling algortihm and propose an optimal solution method in which any particular scheduling algorithm can be used including EDF, Least Laxity First, Rate Monotonic scheduling algorithms [23] and the numerous ones proposed in the literature.

12) We do not consider any particular modulation scheme. For this purpose, instead of formulating the average power consumption and transmit power of a node for a particular modulation scheme as we have done in [14], we propose a generic model in which many modulation schemes reside. We assume that the average power consumption $W_i\left(h_i, b_i, p_i\right)$ and transmit power $W_i^t(b_i, p_i)$ satisfy the following properties:
    a) *$W_i\left(h_i, b_i, p_i\right)$ is a monotonically decreasing function of h_i.*
    b) *$W_i^t(b_i, p_i)$ is a monotonically decreasing function of p_i.*

    Property (a) follows from Eq. (4) and holds for any modulation scheme. Property (b) implies that a lower power consumption can be achieved at the expense of a higher packet error probability keeping the other parameters fixed.

    It can be verified that these properties are satisfied in many modulation schemes including the most common ones QAM (Quadrature Amplitude Modulation) and FSK (Frequency Shift Keying) [16].

13) Our goal is to control the power consumption of the sensor nodes which will enable affordable WNCS deployments by either eliminating battery replacements or prolonging the lifetime of the batteries. To achieve this goal, in the optimization problem formulation, we use a generalized power cost function $f(\{W_i(h_i, b_i, p_i) | i \in [1, N]\})$ to be minimized. The power cost function $f(\{W_i(h_i, b_i, p_i) | i \in [1, N]\})$ satisfies the following property:
    - *$f(\{W_i(h_i, b_i, p_i) | i \in [1, N]\})$ is a monotonically increasing function of $W_i(h_i, b_i, p_i)$ for each and every node i.*

    This generalized power cost function model holds for many commonly used objective functions including the total power consumption of the network [24], the maximum power consumption among the sensor nodes in the network [25] and the log sum of power consumptions of the sensor nodes in the network.

III. Optimization Problem

This section investigates the problem of the joint optimization of control and communication systems. The objective is minimization of a generalized power cost function subject to the stochastic MATI and MAD constraints guaranteeing the stability of the control system and maximum transmit power and schedulability constraints of the wireless communication system.

The optimization problem is formulated as

$$\min_{h_i,b_i,p_i,i\in[1,N]} \quad f(\{W_i(h_i,b_i,p_i)|i\in[1,N]\}) \tag{7a}$$

$$\text{s.t.} \quad \left\lfloor \frac{\Omega}{h_i} \right\rfloor \ln p_i - \ln(1-\delta) \le 0, \quad \forall i \in [1,N], \tag{7b}$$

$$0 < d_i(b_i) \le \min\{\Delta, h_i\}, \quad \forall i \in [1,N], \tag{7c}$$

$$0 < h_i \le \Omega, \quad \forall i \in [1,N], \tag{7d}$$

$$0 < p_i < 1, \quad \forall i \in [1,N], \tag{7e}$$

$$W_i^t(b_i,p_i) \le W^{t,\max}, \quad \forall i \in [1,N], \tag{7f}$$

$$\{d_i(b_i), h_i, \Delta\} \in S^{feasible}, \tag{7g}$$

where N is the number of nodes in the network. Eqs. (7b) and (7c) represent the stochastic MATI and MAD constraints respectively. Eq. (7d) states that the sampling period of the nodes must be less than or equal to the MATI. Eq. (7e) states the lower and upper bounds for the packet error probability. Eq. (7f) represents the maximum transmit power constraint. Finally, Eq. (7g) represents the schedulability constraint. The variables of the problem are $h_i, i \in [1,N]$, the sampling period of the nodes; $b_i, i \in [1,N]$, the constellation size of the nodes; and $p_i, i \in [1,N]$, the packet error probability of the nodes.

This optimization problem is a Mixed-Integer Programming problem thus difficult to solve for the global optimum [26]. In the following, we analyze the optimality conditions for the optimization problem and propose an optimal algorithm.

A. Optimality Analysis

***Lemma** 1:* In the optimal solution, the sampling period h_i^* and packet error probability p_i^* of the nodes are given by

$$\frac{\Omega}{h_i^*} = \frac{\ln(1-\delta)}{\ln p_i^*} = k_i \tag{8}$$

such that the stochastic MATI constraint is satisfied with equality where k_i is a positive integer and equal to the number of transmissions within each Ω.

Proof: We have proven this result for MQAM modulation scheme [14] in which the objective function is the sum of power consumptions of the networks due to the fact that the objective function and transmit power function are monotonically decreasing functions of h_i and p_i. Hence, the result also holds for the optimization problem (7). □

Since h_i and p_i can be represented as functions of a single variable k_i optimally using the expression derived in Lemma 1, we eliminate them from the optimization problem (7) which is then reformulated as

$$\min_{b_i,k_i,i\in[1,N]} \quad f(\{W_i(b_i,k_i)|i\in[1,N]\}) \tag{9a}$$

$$\text{s.t.} \quad 0 < d_i(b_i) \le \min\left\{\Delta, \frac{\Omega}{k_i}\right\}, \quad \forall i \in [1,N], \tag{9b}$$

$$W_i^t(b_i,k_i) \le W^{t,\max}, \quad \forall i \in [1,N], \tag{9c}$$

$$\{d_i(b_i), k_i, \Delta\} \in S^{feasible}, \tag{9d}$$

where the constraints given in Eqs. (7c), (7f) and (7g) correspond to those in Eqs. (7c), (7f) and (7g) respectively and the remaining constraints in the optimization problem (7) are removed due to the additional constraint of k_i being a positive integer. The following lemma expresses the optimal value of k_i in terms of b_i so that the above optimization problem can be formulated with the variable b_i only.

***Lemma** 2:* Suppose that there exists a feasible solution for the optimization problem (9). Then, the optimal value of k_i denoted by k_i^* is the minimum positive integer satisfying Eq. (9c) and can be expressed as a function of b_i.

Proof: Since $f(W_i(h_i,b_i,p_i))$ is a monotonically decreasing function of h_i and p_i, it is a monotonically increasing function of k_i due to Lemma 1. Moreover, minimizing k_i does not shrink the regions for b_i defined by the constraints (9b) and (9d). Hence k_i^* is the minimum positive integer satisfying Eq. (9c) and can therefore be represented as a function of b_i given the transmit power function $W_i^t(b_i,k_i)$. □

Next, we can determine the minimum and maximum b_i values for each sensor node i, denoted by $b_i^{\min}$ and $b_i^{\max}$ respectively, evaluating the constraints given in Eqs. (9b), (9c). Then, using the finding proposed in Lemma 2, the optimization problem can be further simplified as

$$\min_{b_i,i\in[1,N]} \quad f(\{W_i(b_i,k_i^*)|i\in[1,N]\}) \tag{10a}$$

$$\text{s.t.} \quad b_i^{\min} \le b_i \le b_i^{\max}, \quad \forall i \in [1,N], \tag{10b}$$

$$\{d_i(b_i), \Delta\} \in S^{feasible}, \tag{10c}$$

Note that the joint optimization problem given by Eqs. (7) with variables b_i, h_i and p_i has been reduced to a single-variable optimization problem using the optimality relations among the variables. However, since the constellation size b_i is integer, the optimization problem is an IP which is combinatorial in nature. In the following, we propose a fast enumeration algorithm which can solve the problem in reasonable runtime in practice.

B. Optimal Fast Enumeration Algorithm

In order to solve the optimization problem given by Eqs. (10), we propose the Optimal Fast Enumeration Algorithm (OFE), given by Algorithm 1, which is described as follows. First, for each sensor node $i \in [1,N]$, we determine $W_i(b_i,k_i^*)$ for each $b_i \in [b_i^{\min}, b_i^{\max}]$. Let W_{ij} denote the power consumption of node i for constellation size $b_i = j$. We first sort W_{ij} for each node i in increasing order. To use a simpler notation, let W_{i1} be minimum W_{ij}, W_{i2} be the second minimum W_{ij} and so forth. Similarly, let b_{i1} be the constellation size corresponding to minimum power consumption W_{i1}, b_{i2} be the constellation size corresponding to second minimum power consumption W_{i2} and so forth. Let K_{b_i} be the number of feasible b_i values for each node i; i.e., $K_{b_i} = b_i^{\max} - b_i^{\min} + 1$.

Algorithm 1 **Optimum Fast Enumeration Algorithm**

Input: $b_{ij},\ \forall i \in [1, N],\ \forall j \in [1, K_{b_i}]$;
Output: $\mathbf{p}^*$;

1: $f^* = f(b_{1K_{b_1}}, b_{2K_{b_2}}, ..., b_{NK_{b_N}})$;
2: $\mathbf{p} = (b_{11}, b_{21}, ..., b_{N1})$;
3: $deg(\mathbf{p}) = N$;
4: $\mathbf{P} = \{\mathbf{p}\}$;
5: **while** $\mathbf{P} \neq \emptyset$ **do**
6: $\quad$ $\mathbf{P}^+ = \emptyset$;
7: $\quad$ **for** each $\mathbf{p} \in \mathbf{P}$ **do**
8: $\quad\quad$ **if** $f(\mathbf{p}) \leq f^*$ **then**
9: $\quad\quad\quad$ **if isSchedulable(p) then**
10: $\quad\quad\quad\quad$ $\mathbf{p}^* = \mathbf{p}$;
11: $\quad\quad\quad\quad$ $f^* = f(\mathbf{p})$;
12: $\quad\quad\quad$ **else**
13: $\quad\quad\quad\quad$ **for** $j = 1 : deg(\mathbf{p})$ **do**
14: $\quad\quad\quad\quad\quad$ $\mathbf{p}^+ = \mathbf{p}$;
15: $\quad\quad\quad\quad\quad$ $\mathbf{p}^+(N - j + 1) = \mathbf{p}(N - j + 1)^{++}$;
16: $\quad\quad\quad\quad\quad$ $deg(\mathbf{p}^+) = j$;
17: $\quad\quad\quad\quad$ **end for**
18: $\quad\quad\quad\quad$ $\mathbf{P}^+ = \mathbf{P}^+ + \{\mathbf{p}^+\}$;
19: $\quad\quad\quad$ **end if**
20: $\quad\quad$ **end if**
21: $\quad$ **end for**
22: $\quad$ $\mathbf{P} = \mathbf{P}^+$;
23: **end while**

Algorithm starts with the root constellation size vector $(b_{11}, b_{21}, ..., b_{N1})$ corresponding to minimum power consumption for each node in the network (Line 2). Note that each vector $\mathbf{p}$ corresponds to a constellation size vector such that i^{th} element of the vector is the constellation size of the node i. For each vector $\mathbf{p}$, degree of $\mathbf{p}$, denoted by $deg(\mathbf{p})$, is defined to be the number of vectors that vector $\mathbf{p}$ is branched into. Degree of root vector $(b_{11}, b_{21}, ..., b_{N1})$ is set to N (Line 3). Algorithm keeps track of the best solution f^* and f^* is initially set to the objective corresponding to the root vector $(b_{11}, b_{21}, ..., b_{N1})$ (Line 1). Vector set $\mathbf{P}$ is defined to be the set of constellation size vectors to be evaluated in the next iteration of the algorithm and initially contains the vector $(b_{11}, b_{21}, ..., b_{N1})$ (Line 4). For each vector $\mathbf{p}$ in $\mathbf{P}$ (Line 7), the algorithm first determines whether it can improve the best solution so far (Line 8). If the objective corresponding to the vector $\mathbf{p}$ is less than or equal to the best solution so far, the algorithm checks the schedulability of vector $\mathbf{p}$. If $\mathbf{p}$ is schedulable (Line 9), best constellation size vector $\mathbf{p}^*$ is set to $\mathbf{p}$ (Line 10) and best solution is updated to the objective value corresponding to $\mathbf{p}$ (Line 11). Note that schedulable vectors are not branched into new vectors since the objective value of the new vectors branched from a schedulable vector will not be smaller than the objective value of that schedulable node. Otherwise, if $\mathbf{p}$ is not schedulable (Line 12), the algorithm branches the vector $\mathbf{p}$ into $deg(\mathbf{p})$ vectors as follows (Line 14-16). For each $j = 1 : deg(\mathbf{p})$, a vector $\mathbf{p}^+$ is generated by setting the constellation size of node $N - j + 1$ in vector $\mathbf{p}$ to the next constellation size value and degree of $\mathbf{p}^+$ is set to j. Note that newly generated vectors have different degrees. This mechanism guarantees that the algorithm generates a particular vector $\mathbf{p}$ only once and unless there is a schedulable vector, the algorithm generates all possible vectors. Each new vector $\mathbf{p}^+$ is added to set $\mathbf{P}^+$ which is the set of vectors to be evaluated in the next iteration of the algorithm (Line 18). After evaluation of each vector $\mathbf{p}$ in $\mathbf{P}$, $\mathbf{P}$ is equalized to set $\mathbf{P}^+$ (Line 22). Algorithm terminates when there exists no inschedulable vector in $\mathbf{P}$ (Line 5). Since the algorithm can enumerate all possible constellation size vectors and check their schedulability in the worst case, the complexity of OFE is $O(\prod_{i \in [1,N]} K_{b_i} F)$ where F is the complexity required to determine whether a particular constellation size vector yields a feasible schedule using a particular scheduling algorithm. For example, the complexity of schedulability analysis for EDF scheduling algorithm is

$$F = N \sum_{i=1}^{N} \frac{\min\{\frac{c}{1-c} \max_{i \in [1,N]}\{h_i - \Delta\}, \Omega\}}{h_i} \tag{11}$$

where $c = \sum_{i=1}^{N} \frac{d_i}{h_i}$. The details of the schedulability and complexity analysis of EDF scheduling algorithm can be found in [14].

IV. Performance Evaluation

In this section, we evaluate the performance of the proposed optimal solution algorithm, denoted by “OFE”, over the traditional separate design of controller and communication systems denoted by “TS”. In “TS”, the constellation size and sampling period values are predetermined such that the existence of a feasible solution is guaranteed for the worst case scenario without any adjustment to different scenarios. For example, in performance analysis for varying values of MAD values, we select one constellation size value that yields a feasible schedule for all MAD values. Due to space limits, we limit the simulations for the objective of minimizing total power consumption in the network and for the MQAM modulation scheme. EDF scheduling algorithm is used for schedule construction and schedulability analysis.

Simulation results are obtained based on 1000 independent random network topologies where the sensor nodes in the WNCS are uniformly distributed within a circular area of radius r transmitting to a controller located in the center of the area.

σ^2	-174 dBm/Hz	B	10 KHz
$W^{t,\max}$	250 mW	W^c	50mW
$L_i, i \in [1, N]$	100 bits	δ	0.95
N^f	10 dB	G^c	1 (uncoded) [16]

TABLE I: Simulation Parameters

The attenuations of the links are determined considering both small scale statistics that arise mainly from multipath propagation and variations in the communication environment and large scale statistics that arise primarily from the free

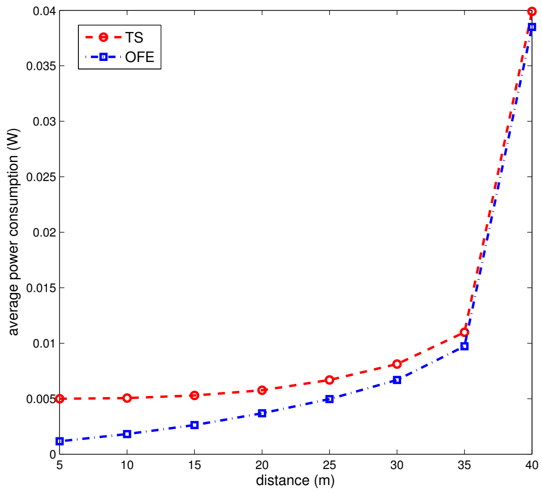

Fig. 3: Average power consumption in a network of 20 nodes at different average distances from the controller where $\Delta = 5$ ms and $\Omega = 100$ ms.

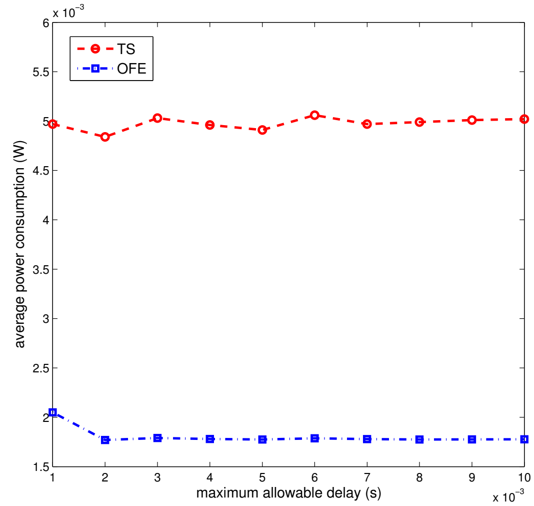

Fig. 4: Average power consumption in a network of 20 nodes for different MAD values where nodes are uniformly distributed within a circular area of radius 10 m and $\Omega = 200$ ms.

space loss and the environment affecting the degree of refraction, diffraction, reflection and absorption. The attenuation of the links considering the large scale statistics is modeled as

$$PL(d) = PL(d_0) + 10\alpha \log(d/d_0) + Z \tag{12}$$

where d is the distance between the sensor node and the controller, $PL(d)$ is the path loss at distance d in decibels, $PL(d_0)$ is the path loss at reference distance $d_0 = 1$ m, α is the path loss exponent [16] and Z is a Gaussian random variable with zero mean and standard deviation σ_z [27]. The path loss model is then extended by the small scale fading that has been modeled by using Rayleigh fading with scale parameter Ω set to the mean power level determined by using Eqn. (12) [27], [28]. The parameters used in the simulations are given in Table-I.

Fig. 3 depicts the average power consumption in a network of 20 nodes at varying average distances from the controller. As distance increases, the transmit power required to compensate for the increasing attenuation increases. The constellation size for the TS algorithm is determined such that there exists a feasible schedule for all distance values. Hence, it is determined considering the maximum distance since feasibility for maximum distance guarantees feasibility for lower distance values. Therefore, the OFE algorithm outperforms the TS algorithm significantly for relatively small average distance values.

Fig. 4 illustrates the average power consumption in a network of 20 nodes for varying MAD values. The MAD constraint determines either the minimum or maximum constellation size depending on the specific modulation scheme used. For MQAM modulation scheme, it specifies the minimum constellation size. As the MAD increases up to a certain value, around 2 ms, the average power consumption decreases since for smaller MAD values, the nodes in the network are forced to choose greater constellation size values, which increases the power consumption dramatically. After that value, increasing MAD value has no effect on the average power consumption due to the fact that the optimal constellation size remains constant although the feasible region expands. Since existence of a feasible solution for minimum MAD value ensures the feasibility of all MAD values, the constellation size for the TS algorithm is determined considering the minimum MAD value. Since the power consumption of a sensor node does not depend on the MAD value, the average power consumption obtained by the TS algorithm remains constant for different MAD values and is dramatically worse than the OFE.

Fig. 5 shows the average power consumption in a network of 20 nodes for different MATI values. The average power consumption decreases as the MATI increases since the power consumption is a decreasing function of MATI. Moreover, as the MATI decreases, the feasible region defined by the schedulability constraint shrinks resulting a higher increase in average power consumption than the functional dependency of power consumption on MATI suggests. Again, the OFE algorithm outperforms the TS algorithm significantly for all MATI values in the specified range. The performance of the TS algorithm is relatively closer to the performance of the OFE algorithm for smaller MATI values since the constellation size is determined considering the minimum MATI value; however, still much worse than the OFE algorithm.

V. Conclusion

A joint optimization framework for the design of communication and control systems in WNCSs is investigated. We have extended our work on joint design of communication and control system in which a joint optimization problem is formulated with the objective of minimizing the power consumption of the network for MQAM modulation scheme. We have generalized the objective of the optimization problem by using a generalized power cost function representing many widely-used power-related objectives including total power consumption of the network, maximum power consumption among the nodes in the network and log sum of the power

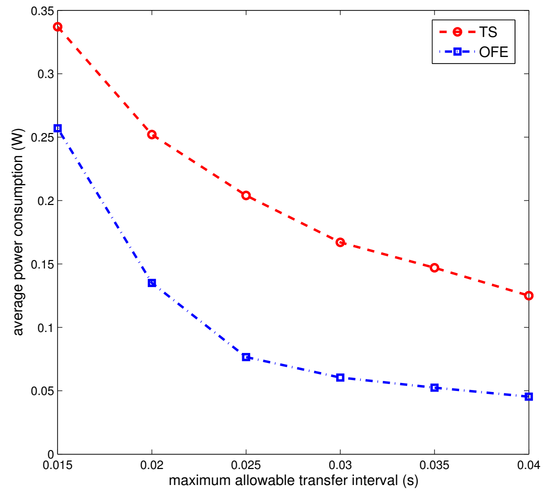

Fig. 5: Average power consumption in a network of 20 nodes for different MATI values where nodes are uniformly distributed within a circular area of radius 10 m and $\Delta = 10$ ms.

consumptions of the nodes in the network. Moreover, we have extended the optimization formulation to be applicable for any modulation scheme that satisfies certain properties including MQAM and MFSK modulation schemes. The optimization problem is formulated as a Mixed-Integer Programming problem which is very difficult to solve for the global optimum. By analyzing the relations among the optimization variables and the sensitivity effects of the optimization variables on the objective of the problem, the optimality conditions are determined allowing us to reformulate the problem as a pure Integer Programming problem. To solve the simplified problem, we propose an optimal fast enumeration algorithm. Simulations show that the proposed optimal solution method performs much better than the algorithm based on separate design of control and communication systems for varying network and control system parameters including distance, MAD and MATI values.

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